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Some Characteristic of α −Fuzzy Orders Relative to α −Fuzzy Subgroups , Normal Subgroup and α −Fuzzy Cyclic Group

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ABSTRACT

In this communication of the paper depicted the $\alpha - FOs$ of a group and then explain the idea of $\alpha - FSG$ and $-FNSG$. More over , we generalized $\alpha - FOs$ relative to α −cyclic group and investigate some characteristic of related algebraic results.

Keywords

Fuzzy Set (FS), fuzzy subset (FSb) ,fuzzy orders (FO), fuzzy group (FG),fuzzy subgroup(FSG), α −fuzzy orders $(\alpha - FO)$, α −fuzzy group$(\alpha - FG)$, α −fuzzy subgroup$(\alpha - FSG)$, α −fuzzy normal subgroup$(\alpha - FNSG)$ and α −Cyclic group.

1. INTRODUCTION

Zadeh L A[9] explored the new idea of fuzzy subsets of a nonempty set in 1965. Abou-Zaid S[1] ,introduced the characteristic fuzzy subgroups of a finite group in 1991. Rosenfeld A[8], explored the new concept of fuzzy groups in 1971. In 1984, Fuzzy Normal subgroups and fuzzy cosets derived from Mukherjee N P and Bhattacharya P[7]. Liu W J[5], described the new idea of fuzzy invariant subgroups and fuzzy ideals in 1982. In 1994, introduced the new notion of fuzzy orders relative to fuzzy subgroups by Jae-Gyeom Kim[6]. In 1981, produced the new concept of fuzzy groups and level subgroups in Das P S[4]. Asaad M[3], developed the new idea in groups and fuzzy subgroups in 1991. In 1988, Some properties of fuzzy groups in explored from the idea is Akgul M[2].

In this research paper arranged as that, section 2 basic fundamental elementary definition and related the results which are through this research article. In section 3, we have define α −fuzzy orders with respect to the α −fuzzy subgroups and α −fuzzy normal subgroups described the some algebraic characteristic results and section 4, we will be introduced the α −fuzzy orders with respect to the α −fuzzy cyclic group and their some generalization results explained.

2. PRELIMINARIES

Definition: 2.1[9]

Let X be a non-empty set . A FSb of the set X is a mapping μ : X→ [0, 1]. **Definition: 2.2[8]**

Let G be a group. A FSb μ of G is a FSG of G if

(i) $\mu(xy) \geq min\{\mu(x), \mu(y)\}$
(ii) $\mu(x^{-1}) \geq \mu(x)$, for all $x, y \in G$.
(iii) From this definition, we clearly have $\mu(x^{-1}) = \mu(x)$, for all $x, y \in G$.

Definition: 2.3[6]

Let G be a group. A FSG μ of G is normal (Invariant) in G if $\mu(xy) = \mu(yx)$ for all $x, y \in G$.

Theorem: 2.4[8]

Let G be a group and let μ be a FSG of G. Then

(i) $\mu(x) \leq \mu(e)$, for all $x, y \in G$.
(ii) $if \mu(xy^{-1}) = \mu(e)$, then $\mu(x) = \mu(y)$

Theorem: 2.5 [7]

Let G be a group and let μ be a FSG of G. Then μ is normal in G if and only if $\mu(y^{-1}xy) = \mu(x)$, for all $x, y \in G$.

Theorem: 2.6 [4]

Let G be a cyclic group of order p^n, where p is a prime number. If μ is a FSG of G, then for all $x, y \in G$.

(i) If $O(x) > O(y), then\ \mu(x) \leq \mu(y)$.
(ii) If $O(x) = O(y)$, then $\mu(x) = \mu(y)$.

Theorem: 2.7 [2]

Let G be a finite group and let μ be a FSG of G. Then

(i) $\mu(x^K) \geq \mu(x)$ for any integer K and for all $x \in G$.
(ii) If $O(x)/O(y)$, then $\mu(y) \leq \mu(x)$ for $x, y \in \langle Z \rangle$, where $z \in G$.
(iii) If $(O(x), K) = 1$, then $\mu(x^k) = \mu(x), where\ k \in Z\ and\ x \in G$.

Theorem: 2.8

Let G be a group. For $x, y, z \in G$, we have

(i) If $x^m = e$, then $O(x)/m$, where $m \in Z$.
(ii) $O(x^m) = O(x)/(O(x), m)$, where $m \in Z$.
(iii) If $(O(x), O(y)) = 1$ and $xy = yx$, then $O(xy) = O(x) \times O(y)$.
(iv) If $z = y^{-1}xy$, then $O(z) = O(x)$.
(v) If $O(z) = mn$ with $(m, n) = 1$, then $z = xy = yx$ for some $x, y \in G$ with $O(x) = m$ and $O(y) = n$. Further, such an expression for z is unique.

Definition: 2.9 [6]

Let μ be a FSG of a group G. For a given $x \in G$, the least positive integer n such that $\mu(x^n) = \mu(e)$ is the FO of x with respect to μ [briefly, $FO_\mu(x)$]. If no such n exists, x is of infinite FO with respect to μ.

3. SOME CHARACTERISTIC OF $\alpha - FOs$ RELATIVE TO $\alpha - FSG$

Definition: 3.1

Let A^α be a $\alpha - FSG$ of a group G. For a given $\theta \in G$, the least positive integer n such that $A^\alpha(\theta^n) = A^\alpha(e)$ is the $\alpha - FO$ of θ with respect to A^α [briefly, $FO_{A\alpha}(\theta)$]. If no such n exists, θ is of infinite $\alpha - FO$ with respect to A^α.

$\therefore O(\theta)$ and $O(\varphi)$ does not imply that of $FO_{A\alpha}(\theta)$ and $FO_{A\alpha}(\varphi)$,

Example: 3.1.1

Let $G = \{a, b/a^2 = b^2 = (ab)^2 = e\}$ be the Klein four-group. Define a $\alpha - FSG$ A^α of G by $A^\alpha(e) = A^\alpha(ab) = t_o$ and $A^\alpha(a) = A^\alpha(b) = t_1$, where $t_o > t_1$. Clearly, $O(a) = O(ab) = 2$, but

$FO_{A\alpha}(a) = 2$ and $FO_{A\alpha}(ab) = 1$.

Proposition: 3.2

Let A^α be a $\alpha - FSG$ of a group G. For $\theta \in G$, if $A^\alpha(\theta^m) = A^\alpha(e)$ for some integer m, then $FO_{A\alpha}(\theta)/m$. Proof:

Let $FO_{A^\alpha}(\theta) = n$. If $\exists$ integers $s\ and\ t$: $m = ns + t$, where $0 \leq t < n$. Then, $A^\alpha(\theta^t) = A^\alpha(\theta^{m-ns}) = A^\alpha(\theta^m(\theta^n)^{-s}) \geq min\{A^\alpha(\theta^m), A^\alpha((\theta^n)^{-s})\}$

$\geq min\{A^\alpha(e), A^\alpha(\theta^n)\} = min\{A^\alpha(e), A^\alpha(e)\} = A^\alpha(e)$.

Hence $t = 0$, by the choice of n. If $O(\theta)$ is finite then $FO_{A\alpha}(\theta)$ is clearly finite for all $\alpha - FSG$ A^α of G. If $O(\theta)$ is infinite, then for each positive integer n, $\exists$ a $\alpha - FSG$ $A^\alpha{}_n$ of G : $FO_{A\alpha n}(\theta) = n$ as follows.

■

Example: 3.2.1

Let θ be an element of infinite order in the group G. For each positive integer n, define the $\alpha - FSG$ $A_{\alpha n}$ of G by $A_{\alpha n}(\varphi) = \{t_o \text{ if } \varphi \in \langle\theta_n\rangle, t_1 \text{ otherwise}$,

Where $t_o > t_1$. Clearly, $FO_{A_{\alpha n}}(\theta) = n$. ■

Corollary: 3.2.2

Let A^α be a $\alpha - FSG$ of a group G. Then $FO_{A\alpha}(\theta)/\ O(\theta)$ for all $\theta \in G$.

Proposition: 3.3

Let A^α be a $\alpha - FSG$ of a group G, and let θ and φ be elements of G : $(FO_{A\alpha}(\theta), FO_{A\alpha}(\varphi)) = 1$ and $\theta\varphi = \varphi\theta$. If $A^\alpha(\theta\varphi) = A^\alpha(e)$, then $A^\alpha(\theta) = A^\alpha(\varphi) = A^\alpha(e)$. Proof:

Let $FO_{A\alpha}(\theta) = n$ and $FO_{A\alpha}(\varphi) = m$. Then $A^\alpha(e) = A^\alpha(\theta\varphi) \leq A^\alpha((\theta\varphi)^m) = A^\alpha(\theta^m\varphi^m)$.
Thus $A^\alpha(\theta^m) = A^\alpha(\varphi^m)$=$A^\alpha(e)$. Therefore, n/m, by pro.. (3.2).
But $(n, m) = 1$. Thus $n = 1$, i.e., $A^\alpha(\theta) = A^\alpha(e)$.
Hence $A^\alpha(\varphi) = A^\alpha(\theta) = A^\alpha(e)$. ■

Within the proposition, although A^α is normal, the belief $\theta\varphi = \varphi\theta$ may not be omitted . **Corollary: 3.3.1**

Let A^α be a $\alpha - FSG$ of a group G, and let θ and φ be elements of G such that $(O(\theta), O(\varphi)) = 1$ and $\theta\varphi = \varphi\theta$. If $A^\alpha(\theta\varphi) = A^\alpha(e)$, then $A^\alpha(\theta) = A^\alpha$ (y)=$A^\alpha(e)$.

Neither the assumption $(FO_{A\alpha}(\theta), FO_{A\alpha}(\varphi)) = 1$ in pro…(3.3) nor the assumption $(O(\theta), O(\varphi)) = 1$ in corollary 3.3.1 can be omitted. In fact, in example 3.1.1 $A^\alpha(a) = A^\alpha(b) \neq A^\alpha(e)$, but $FO_{A\alpha}(a)$=$FO_{A\alpha}(b) = O(a) = O(b) = 2$. ■

Theorem: 3.4

Let A^α be a $\alpha - FSG$ of a group G. Let $FO_{A\alpha}(\theta) = n$, where $\theta \in G$. If m is an integer with $d = (m, n)$, then $FO_{A\alpha}(\theta^m) = n/d$. Proof:

Let $FO_{A\alpha}(\theta^m) = t$. First we have

$$A^\alpha((\theta^m)^{\frac{n}{d}}) = A^\alpha(\theta^{nk}) \text{ for some integer } k$$
$$\geq A^\alpha(\theta^n) = A^\alpha(e).$$

Thus $t/n/d$ by pro..(3.2). Because $d = (m, n)$, $\exists$ integer i and j: $ni + mj = d$.
We the have

$$A_\alpha(\theta^{td}) = A^\alpha(\theta^{t(ni+mj)}) = A^\alpha(\theta_{nti}\theta_{mtj})$$
$$\geq min\{A^\alpha((\theta^n)^{ti}), A^\alpha(((\theta^m)^t)^j)\}$$
$$\geq min\{A^\alpha(\theta^n), A^\alpha((\theta^m)^t)\}$$
$$= min\{A^\alpha(e), A^\alpha(e)\} = A^\alpha(e).$$

This implies that n/td i.e., $n/d/t$. Consequently , $t = n/d$. ■

Proposition: 3.5

Let A^α be a $\alpha - FSG$ G. Let $FO_{A\alpha}(\theta) = n$, where $\theta \in G$. If m is an integer with $(n, m) = 1$, then $A^\alpha(\theta^m) = A^\alpha(\theta)$.

Proof:

Because $(n, m) = 1$, $\exists$ integers s and t : $ns + mt = 1$.

We then have

$$A^\alpha(\theta) = A^\alpha(\theta^{ns+mt}) = A^\alpha((\theta^n)^s)(\theta^m)^t))$$
$$\geq min\{A^\alpha(\theta^n)^s), A^\alpha(\theta^m)^t)\}$$
$$\geq min\{A^\alpha(\theta^n), A^\alpha(\theta^m)\}$$
$$= min\{A^\alpha(e), A^\alpha(\theta^m)\}$$
$$= A^\alpha(\theta^m) \geq A^\alpha(\theta).$$ ■

Theorem: 3.6

Let A^α be a $\alpha - FSG$ of a group G. Let $FO_{A\alpha}(\theta) = n$, where $\theta \in G$. If $i \equiv j (mod\ n)$, where $i, j \in Z$, then $FO_{A\alpha}(\theta^i) = FO_{A\alpha}(\theta^j)$. Proof:

Let $FO_{A\alpha}(\theta^i) = t$ and $FO_{A\alpha}(\theta^j) = s$. By the assumption, $i = j + nk$ for some integer K.

Now, $A^\alpha((\theta^i)^s) = A^\alpha((\theta^{j+nk})^s) = A^\alpha\ ((\theta^j)^s(\theta^n)^{ks}))$

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$\geq min\{A^{\alpha}(\theta^{j})^{s}), A^{\alpha}(\theta^{n})^{ks})\}$

$\geq min\{A^{\alpha}(e), A^{\alpha}(\theta^{n})\}$

$= min\{A^{\alpha}(e), A^{\alpha}(e)\}$,

And so t/s. Similarly, s/t. Thus we have $t = s$. ■

Theorem: 3.7

Let A^{α} be a $\alpha - FSG$ of a group G, and let θ and φ be elements of $G : \theta\varphi = \varphi\theta$ and $(FO_{A\alpha}(\theta), FO_{A\alpha}(\varphi)) = 1$. Then $FO_{A\alpha}(\theta\varphi) = FO_{A\alpha}(\theta) \times FO_{A\alpha}(\varphi)$. Proof:

Let $FO_{A\alpha}(\theta\varphi) = n, FO_{A\alpha}(\theta) = s$ and $FO_{A\alpha}(\varphi) = t$.

Then $A\alpha((\theta\varphi)_{st}) = A\alpha(\theta_{st}\varphi_{st})$

$\geq min\{A^{\alpha}((\theta^{s})^{t}), A^{\alpha}((\varphi^{t})^{s})\}$

$\geq min\{A^{\alpha}(\theta^{s}), A^{\alpha}(\varphi^{t})\} =$

$min\{A^{\alpha}(e), A^{\alpha}(e)\} = A^{\alpha}(e)$. Thus n/st ,Now $A^{\alpha}(e) = A^{\alpha}((\theta\varphi)^{n}) =$

$A^{\alpha}(\theta^{n}\varphi^{n})$. Besides, $(FO_{A\alpha}(\theta^{n}), FO_{A\alpha}(\varphi^{n})) = 1$.

$\therefore A^{\alpha}(\theta^{n}) = A^{\alpha}(\varphi^{n}) = A^{\alpha}(e)$ both s and t divide n .

$\therefore st/n$, because $(s, t) = 1 \quad \Rightarrow n = st$. ■

Corollary: 3.7.1

Let A^{α} be a $\alpha - FSG$ of a group G, and let θ and φ be elements of $G : \theta\varphi = \varphi\theta$ and $(O(\theta), O(\varphi)) = 1$. Then $FO_{A\alpha}(\theta\varphi) = FO_{A\alpha}(\theta) \times FO_{A\alpha}(\varphi)$.

$\because$ supposing A^{α} is normal subgroup, the assumption $\theta\varphi = \varphi\theta$ may not be omitted.

Example: 3.7.2

Define a $\alpha - FNSG \ A^{\alpha}$ of the symmetric group S_4

$$A_{\alpha}(\theta) = \{ t_o \ if \ \theta = e, \quad t_1 \ otherwise,$$

Where $t_o > t_1$. Now, let $\theta = (1 \ 2)$ and $\varphi = (2 \ 3 \ 4)$. Then $FO_{A\alpha}(\theta) = 2, FO_{A\alpha}(\varphi) = 3$,

$FO_{A\alpha}(\theta\varphi) = FO_{A\alpha}(\varphi\theta) = 4$, and $\theta\varphi \neq \varphi\theta$. ■

Theorem: 3.8

Let A^{α} be a $\alpha - FSG$ of a group G. For $z \in G$, if $FO_{A\alpha}(z) = nm$ with $(n, m) = 1$, then $\exists \ \theta$ and φ in $G : z = \theta\varphi = \varphi\theta$, $FO_{A\alpha}(\theta) = m$ and $FO_{A\alpha}(\varphi) = n$. Furthermore explain for z is unique in the sense of α $-$fuzzy grades, i.e., if (θ, φ) and (θ_1, φ_1) are such pairs, then $A^{\alpha}(\theta) = A^{\alpha}(\theta_1)$ and $A^{\alpha}(\varphi) = A^{\alpha}(\varphi_1)$. Proof

Because $(m, n) = 1$, $\exists$ integers s and $t : ms + nt = 1$.

Here $(m, t) = (n, s) = 1$. Let $\theta = z^{nt}$ and $\varphi = z^{ms}$. Then $Z = \theta\varphi = \varphi\theta$, and by theorem 3.4, $FO_{A\alpha}(\theta) = FO_{A\alpha}(Z^{nt}) = m$ and $FO_{A\alpha}(\varphi) = FO_{A\alpha}(Z^{ms}) = n$. This proves the existence of θ and φ. Let (θ, φ) and (θ_1, φ_1) be pairs satisfied. since $FO_{A\alpha}(\theta) = FO_{A\alpha}(\theta_1) = m$ and $FO_{A\alpha}(\varphi) = FO_{A\alpha}(\varphi_1) = n, \quad \Rightarrow A\alpha(\theta) = A\alpha(\theta_{1-ms}) = A\alpha(\theta_{nt}) = A\alpha(\theta_{nt}\varphi_{nt}) = A\alpha((\theta\varphi)_{nt})$

$= A\alpha((\theta_1\varphi_1)_{nt}) = A\alpha(\theta_{1nt}\varphi_{1nt}) = A\alpha(\theta_{1nt})$

$= A\alpha(\theta_{11-ms}) = A\alpha(\theta_1)$.

Similarly, $A^{\alpha}(\varphi) = A^{\alpha}(\varphi_1)$.

This proves the uniqueness of (θ, φ). ■

Theorem: 3.9

Let A^{α} be a $\alpha - FNSG$ of a group G. Then $FO_{A\alpha}(\theta) = FO_{A\alpha}(\varphi^{-1}\theta\varphi)$ for all $\theta, \varphi \in G$. Proof:

Let $\theta, \varphi \in G$, then we have $A^{\alpha}(\theta^{n}) = A^{\alpha}(\varphi^{-1}\theta^{n}\varphi) = A^{\alpha}((\varphi^{-1}\theta\varphi)^{n})$ for all $n \in Z$.

Thus $FO_{A\alpha}(\theta) = FO_{A\alpha}(\varphi^{-1}\theta\varphi)$. ■

$\therefore A^{\alpha}$ is not normal in G.

Example: 3.9.1

Let D_3=$\{a, b/a^{3} = b^{3} = e, ba = a^{2}b\}$ be the group with 6 elements. Define a $\alpha - FSG \ A^{\alpha}$ of D_3 by

$$A_{\alpha}(\theta) = \{ t_o \ if \ \theta \in \langle b \rangle, \quad t_1 \ otherwise$$

Where $t_o > t_1$. Then $a^{-1}ba \notin \langle b \rangle$, and so $FO_{A\alpha}(b) = 1 \neq FO_{A\alpha}(a^{-1}ba)$. ■

4. ALGEBRAIC PROPERTIES OF $\alpha - FOs$ IN A CYCLIC GROUP

Lemma: 4.1

Let A^α be a $\alpha - FSG$ of a cyclic group G and let a and b be any two generators of G. Then $FO_{A^\alpha}(a) = FO_{A^\alpha}(b)$. Proof

We have apply for Theroem..(3.4).

Theorem: 4.2

Let A^α be a $\alpha - FSG$ of a cyclic group G of finite order n. Then, $\forall\ \theta, \varphi \in G$;

(i) If $O(\theta) = O(\varphi)$, then $FO_{A^\alpha}(\theta) = FO_{A^\alpha}(\varphi)$. (ii) If $O(\theta)/O(\varphi)$, then $FO_{A^\alpha}(\theta)/FO_{A^\alpha}(\varphi)$.

(iii) If $O(\theta) > O(\varphi)$, then $FO_{A^\alpha}(\theta) \geq FO_{A^\alpha}(\varphi)$.

Proof

Let $G = \langle a \rangle$. Let $\theta = a^s$, $\varphi = a^t$, and $FO_{A^\alpha}(a) = m$.

m is a specific generator a of G.

Then $O(\theta) = n/(s, n)$, $FO_{A^\alpha}(\theta) = m/(s, m)$, $FO_{A^\alpha}(\varphi) = m/(t, m)$ and m/n, (i) Follows from (ii).

(ii) If $O(\theta)/O(\varphi)$, then $(t, n)/(s, n)$, and so $(t, m)/(s, m)$, because m/n. Thus $FO_{A^\alpha}(\theta)/ FO_{A^\alpha}(\varphi)$.

(iii) If $O(\theta) > O(\varphi)$, the $(s, n) < (t, n)$, and so $(s, m) \leq (t, m)$, because m/n. Thus $FO_{A^\alpha}(\theta) \geq FO_{A^\alpha}(\varphi)$. ■

Theorem: 4.3

Let A^α be a $\alpha - FSG$ of a cyclic group G of finite order. Then , $\forall\ \theta, \varphi \in G$:

(i) If $FO_{A^\alpha}(\theta) = FO_{A^\alpha}(\varphi)$, then $A^\alpha(\theta) = A^\alpha(\varphi)$.

(ii) If $FO_{A^\alpha}(\theta)/FO_{A^\alpha}(\varphi)$, then $A^\alpha(\theta) \geq A^\alpha(\varphi)$. Proof

Let $G = \langle a \rangle$. Let $\theta = a^s$, $\varphi = a^t$, and $FO_{A^\alpha}(a) = m$.

m is a specific generator a of G.

Then $FO_{A^\alpha}(\theta) = m/(s, m)$ and $FO_{A^\alpha}(\varphi) = m/(t, m)$,

Let $s = h(s, m)$, $t = i(t, m)$ and $m = j(t, m) = k(s, m)$ for some $h, i, j, k \in Z$.

If $FO_{A^\alpha}(\theta)/FO_{A^\alpha}(\varphi)$, then $(t, m)/(s, m)$. So $t/si = h(s, m)i$ and $m/sj = h(s, m)j$,

$\Rightarrow A^\alpha(\theta) = A^\alpha(a^s)$

$= A^\alpha(a^{s(iv+jw)})$ for some $u, w \in Z$, since $(i, j) = 1$

$= A\alpha(a_{siv}a_{sjw}) \geq min\{A\alpha(a_{siv}), A\alpha(a_{sjw})\}$

$\geq min\{A^\alpha(a^t), A^\alpha(a^m)\} = min\{A^\alpha(\varphi), A^\alpha(e)\} = A^\alpha(\varphi)$. ■

Corollary: 4.3.1

Let A^α be a $\alpha - FSG$ of a cyclic group G of finite order. Then, $\forall\ \theta, \varphi \in G$:

(i) If $O(\theta) = O(\varphi)$, $then\ A^\alpha(\theta) = A^\alpha(\varphi)$.

(ii) If $O(\theta)/O(\varphi)$, $then\ A^\alpha(\theta) \geq A^\alpha(\varphi)$. ■

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