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BINARY GENERALIZED STAR PRE CLOSED SETS IN BINARY TOPOLOGICAL SPACES

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ABSTRACT. We introduce and study a new class of the set binary $g^{\star}p$-closed sets and some characterize the relations between them and the related properties are investigate with suitable examples in main aim of this paper.

1. Introduction and Preliminaries

In 1970 Levine [5] gives the concept and properties of generalized closed (briefly g-closed) sets and the complement of g-closed set is said to be g-open set. Njasted [14] introduced and studied the concept of α-sets. Later these sets are called as α-open sets in 1983. Mashhours et.al [8] introduced and studied the concept of α-closed sets, α-closure of set, α-continuous functions, α-open functions and α-closed functions

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in topological spaces. Maki et.al [6, 7] introduced and studied generalized α-closed sets and α-generalized closed sets. In 2011, S.Nithyanantha Jothi and P.Thangavelu [9] introduced topology between two sets and also studied some of their properties. Topology between two sets is the binary structure from X to Y which is defined to be the ordered pairs (A, B) where $A \subseteq X$ and $B \subseteq Y$. In this paper, We introduce and study a new class of the set binary g^*p-closed sets and some characterize the relations between them and the related properties are investigate with suitable examples.

Throughout this paper, (X, Y) denote binary topological spaces $(X, Y, \mathcal{M})$.

Let X and Y be any two nonempty sets. A binary topology [9] from X to Y is a binary structure $\mathcal{M} \subseteq \mathcal{P}(X) \times \mathcal{P}(Y)$ that satisfies the axioms namely

(1) (φ, φ) and $(X, Y) \in \mathcal{M}$,
(2) $(A_1 \cap A_2, B_1 \cap B_2) \in \mathcal{M}$ whenever $(A_1, B_1) \in \mathcal{M}$ and $(A_2, B_2) \in \mathcal{M}$, and
(3) If $\{(A_\alpha, B_\alpha) : \alpha \in \delta\}$ is a family of members of $\mathcal{M}$, then $(\bigcup_{\alpha\in\delta} A_\alpha, \bigcup_{\alpha\in\delta} B_\alpha) \in \mathcal{M}$.

If $\mathcal{M}$ is a binary topology from X to Y then the triplet $(X, Y, \mathcal{M})$ is called a binary topological space and the members of $\mathcal{M}$ are called the binary open subsets of the binary topological space $(X, Y, \mathcal{M})$. The elements of $X \times Y$ are called the binary points of the binary topological space $(X, Y, \mathcal{M})$. If $Y = X$ then $\mathcal{M}$ is called a binary topology on X in which case we write $(X, \mathcal{M})$ as a binary topological space.

Definition 1.1. [9] *Let X and Y be any two nonempty sets and let (A, B) and $(C, D) \in \mathcal{P}(X) \times \mathcal{P}(Y)$. We say that $(A, B) \subseteq (C, D)$ if $A \subseteq C$ and $B \subseteq D$.*

Definition 1.2. [9] *Let $(X, Y, \mathcal{M})$ be a binary topological space and $A \subseteq X$, $B \subseteq Y$. Then (A, B) is called binary closed in $(X, Y, \mathcal{M})$ if $(X \setminus A, Y \setminus B) \in \mathcal{M}$.*

Proposition 1.3. [9] *Let* $(X, Y, \mathcal{M})$ *be a binary topological space and* $(A, B) \subseteq (X, Y)$.

Let $(A, B)^{1*} = \cap\{A_\alpha : (A_\alpha, B_\alpha)$ *is binary closed and* $(A, B) \subseteq (A_\alpha, B_\alpha)\}$ *and* $(A, B)^{2*} = \cap\{B_\alpha : (A_\alpha, B_\alpha)$ *is binary closed and* $(A, B) \subseteq (A_\alpha, B_\alpha)\}$. *Then* $((A, B)^{1*}, (A, B)^{2*})$ *is binary closed and* $(A, B) \subseteq ((A, B)^{1*}, (A, B)^{2*})$.

Proposition 1.4. [9] *Let* $(X, Y, \mathcal{M})$ *be a binary topological space and* $(A, B) \subseteq (X, Y)$. *Let* $(A, B)^{1*} = \cup\{A_\alpha : (A_\alpha, B_\alpha)$ *is binary open and* $(A_\alpha, B_\alpha) \subseteq (A, B)\}$ *and* $(A, B)^{2*} = \cup\{B_\alpha : (A_\alpha, B_\alpha)$ *is binary open and* $(A_\alpha, B_\alpha) \subseteq (A, B)\}$.

Definition 1.5. [9] *The ordered pair* $((A, B)^{1*}, (A, B)^{2*})$ *is called the binary closure of* (A, B), *denoted by* $b\text{-}cl(A, B)$ *in the binary space* $(X, Y, \mathcal{M})$ *where* $(A, B) \subseteq (X, Y)$.

Definition 1.6. [9] *The ordered pair* $((A, B)^{1*}, (A, B)^{2*})$ *defined in proposition 1.4 is called the binary interior of of* (A, B), *denoted by* $b\text{-}int(A, B)$. *Here* $((A, B)^{1*}, (A, B)^{2*})$ *is binary open and* $((A, B)^{1*}, (A, B)^{2*}) \subseteq (A, B)$.

Definition 1.7. [9] *Let* $(X, Y, \mathcal{M})$ *be a binary topological space and let* $(x, y) \subseteq (X, Y)$. *The binary open set* (A, B) *is said to be a binary neighbourhood of* (x, y) *if* $x \in A$ *and* $y \in B$.

Proposition 1.8. [9] *Let* $(A, B) \subseteq (C, D) \subseteq (X, Y)$ *and* $(X, Y, \mathcal{M})$ *be a binary topological space. Then, the following statements hold:*

(1) $b\text{-}int(A, B) \subseteq (A, B)$.
(2) *If* (A, B) *is binary open, then* $b\text{-}int(A, B) = (A, B)$.
(3) $b\text{-}int(A, B) \subseteq b\text{-}int(C, D)$.
(4) $b\text{-}int(b\text{-}int(A, B)) = b\text{-}int(A, B)$.
(5) $(A, B) \subseteq b\text{-}cl(A, B)$.

(6) *If* (A, B) *is binary closed, then* $b\text{-}cl(A, B) = (A, B)$.

(7) $b\text{-}cl(A, B) \subseteq b\text{-}cl(C, D)$.

(8) $b\text{-}cl(b\text{-}cl(A, B)) = b\text{-}cl(A, B)$.

Definition 1.9. *A subset* (A, B) *of a binary topological space* $(X, Y, \mathcal{M})$ *is called*

(1) *a binary semi open set* [13] *if* $(A, B) \subseteq b\text{-}cl(b\text{-}int(A, B))$.

(2) *a binary pre open set* [3] *if* $(A, B) \subseteq b\text{-}int(b\text{-}cl(A, B))$,

(3) *a binary regular open set* [12] *if* $(A, B) = b\text{-}int(b\text{-}cl(A, B))$.

Definition 1.10. *A subset* (A, B) *of a binary topological space* $(X, Y, \mathcal{M})$ *is called*

(1) *a binary g-closed set* [10] *if* $b\text{-}cl(A, B) \subseteq (U, V)$ *whenever* $(A, B) \subseteq (U, V)$ *and* (U, V) *is binary open.*

(2) *a binary gp-closed set* [4] *if* $b\text{-}pcl(A, B) \subseteq (U, V)$ *whenever* $(A, B) \subseteq (U, V)$ *and* (U, V) *is binary open.*

Definition 1.11. [2] *Let* (A, B) *be a subset of a binary topological space* (X, Y). *Then* (A, B) *is called a binary* $g^{\star}$*-closed set if* $b\text{-}cl(A, B) \subseteq (P, Q)$ *whenever* $(A, B) \subseteq (P, Q)$ *and* (P, Q) *is binary g-open in* (X, Y).

2. Binary $g^{\star}p$-closed sets

Definition 2.1. *Let* $(X, Y, \mathcal{M})$ *be a binary topological space. A subset* (A, B) *of* $(X, Y, \mathcal{M})$ *is called binary generalized star pre closed set (briefly binary* $g^{\star}p$*-closed) if* $bp\text{-}cl(A, B) \subseteq (U, V)$ *where* $(A, B) \subseteq (U, V)$ *and* (U, V) *is binary g-open.*

Theorem 2.2. *If* (A, B) *is binary closed set in* $(X, Y, \mathcal{M})$, *then it is a binary* $g^{\star}p$*-closed set but not the converse.*

Proof. Let (A, B) be a binary closed set of (X, Y) and $(A, B) \subseteq (U, V)$ where (U, V) is binary g-open in (X, Y). Since (A, B) is binary closed, we have $b\text{-}cl(A, B) =$

$(A, B) \subseteq (U, V)$. That is $b\text{-}cl(A, B) \subseteq (U, V)$. Also $bp\text{-}cl(A, B) \subseteq b\text{-}cl(A, B)$ implies $bp\text{-}cl(A, B) \subseteq (U, V)$, where (U, V) is binary g-open in (X, Y). Therefore (A, B) is a binary $g^{\star}p$-closed set.

Example 2.3. *Let* $X = \{a, b\}$, $Y = \{1, 2\}$ *and* $\mathrm{M} = \{(\varphi, \varphi), (\varphi, \{1\}), (\{a\}, \{1\}), (X, Y)\}$. *Then the set* $(\{a\}, \{2\})$ *is binary* $g^{\star}p$*-closed but not binary closed set.*

Theorem 2.4. *If* (A, B) *is binary* g*-closed in* (X, Y, M)*, then it is binary* $g^{\star}p$*-closed set but not the converse.*

Proof. Let (A, B) be a binary g-closed set. Then $b\text{-}cl(A, B) \subseteq (U, V)$ whenever $(A, B) \subseteq (U, V)$ and (U, V) is binary open in (X, Y), since every binary open set is binary g-open set. So (U, V) is binary g-open set in (X, Y). We have $bp\text{-}cl(A, B) \subseteq b\text{-}cl(A, B)$ which implies $bp\text{-}cl(A, B) \subseteq (U, V)$, $(A, B) \subseteq (U, V)$, (U, V) is binary g-open in (X, Y). Hence (A, B) is binary $g^{\star}p$-closed set.

Example 2.5. *Let* $X = \{a, b\}$, $Y = \{1, 2\}$ *and* $\mathrm{M} = \{(\varphi, \varphi), (\varphi, \{1\}), (\{a\}, \{1\}), (\{b\}, \{1\}), (X, \{1\}), (X, Y)\}$. *Then the set* $(\{b\}, \varphi)$ *is binary* $g^{\star}p$*-closed but not a binary* g*-closed set.*

Theorem 2.6. *If* (A, B) *is binary* $g^{\star}$*-closed in* (X, Y, M)*, then it is binary* $g^{\star}p$*-closed set but the converse is not true.*

Proof. Let (A, B) be a binary $g^{\star}$-closed set of (X, Y) and $(A, B) \subseteq (U, V)$, where (U, V) is binary g-open in (X, Y). Since (A, B) is binary $g^{\star}$-closed we have $b\text{-}cl(A, B) = (A, B)$. So $(A, B) \subseteq (U, V)$ implies $b\text{-}cl(A, B) \subseteq (X, Y)$. But $bp\text{-}cl(A, B) \subseteq b\text{-}cl(A, B)$ implies $bp\text{-}cl(A, B) \subseteq (U, V)$, $(A, B) \subseteq (U, V)$, (U, V) is binary g-open in (X, Y). Therefore (A, B) is binary $g^{\star}p$-closed set.

Example 2.7. *In Example 2.5, the set* (X, φ) *is binary* $g^{\star}p$*-closed set but not a binary* $g^{\star}$*-closed set.*

Theorem 2.8. *If* (A, B) *is binary gp-closed in* (X, Y, M)*, then it is binary* $g^{\star}p$*-closed set.*

Proof. Let (A, B) be a binary gp-closed set of (X, Y) and $(A, B) \subseteq (U, V)$, where (U, V) is binary open in (X, Y). But every binary open sets is binary g-open set. This implies (U, V) is binary g-open in (X, Y). So $bp\text{-}cl(A, B) \subseteq (U, V)$, $(A, B) \subseteq (U, V)$, (U, V) is binary g-open in (X, Y). Therefore (A, B) is binary $g^{\star}p$-closed set.

Theorem 2.9. *The union of two binary* $g^{\star}p$*-closed sets in* (X, Y, M) *is also a binary* $g^{\star}p$*-closed set in* (X, T, M)*.*

Proof. Let (A, B) and (C, D) be two binary $g^{\star}p$-closed sets in (X, Y, M). Let (U, V) be a binary g-open set in (X, Y), such that $(A, B) \subseteq (U, V)$ and $(C, D) \subseteq (U, V)$. Then we have $((A, B) \cup (C, D)) \subseteq (U, V)$. Since (A, B) and (C, D) are binary $g^{\star}p$-closed in (X, Y, M). This implies $bp\text{-}cl(A, B) \subseteq (U, V)$ and $bp\text{-}cl(C, D) \subseteq (U, V)$. Now $bp\text{-}cl((A, B) \cup (C, D)) = bp\text{-}cl(A, B) \cup bp\text{-}cl(C, D) \subseteq (U, V)$. Thus we have $bp\text{-}cl((A, B) \cup (C, D)) \subseteq (U, V)$, whenever $((A, B) \cup (C, D)) \subseteq (U, V)$, where (U, V) is binary g-open set in (X, Y, M). This implies $(A, B) \cup (C, D)$ is a binary $g^{\star}p$-closed set in (X, Y, M).

Remark 2.10. *The intersection of two binary* $g^{\star}p$*-closed sets in* (X, Y, M) *is also a binary* $g^{\star}p$*-closed set in* (X, Y, M) *as seen from the following example.*

Example 2.11. *Let* $X = \{a, b\}$, $Y = \{1, 2\}$ *and* $\mathrm{M} = \{(\varphi, \varphi), (\varphi, \{1\}), (\varphi, \{2\}), (\varphi, Y), (\{a\}, \{1\}), (\{a\}, Y), (\{b\}, \{2\}), (\{b\}, Y), (X, Y)\}$. *Then the binary* $g^{\star}p$*-closed sets are* $\{(\varphi, \varphi), (\{a\}, \varphi), (\{a\}, \{1\}), (\{b\}, \varphi), (\{b\}, \{2\}), (X, \varphi), (X, \{1\}), (X, \{2\}),$

$(X, Y)\}$. Let $A = (\{a\}, \{1\})$ and $B = (X, \varphi)$ are binary $g^{\star}p$-closed sets. Then $A \cap B = (\{a\}, \{1\}) \cap (X, \varphi) = (\{a\}, \varphi)$ is also binary $g^{\star}p$-closed set.

Theorem 2.12. *Let (A, B) be a binary $g^{\star}p$-closed subset of (X, Y, M). If $(A, B) \subseteq (C, D) \subseteq bp\text{-}cl(A, B)$, then (C, D) is also a binary $g^{\star}p$-closed subset of (X, Y, M).*

Proof. Let (U, V) be a binary g-open set of a binary $g^{\star}p$-closed subset of M such that $(C, D) \subseteq (U, V)$, as $(A, B) \subseteq (C, D)$, we have $(A, B) \subseteq (U, V)$. As (A, B) is binary $g^{\star}p$-closed set, $bp\text{-}cl(A, B) \subseteq (U, V)$, Given $(C, D) \subseteq bp\text{-}cl(A, B)$. We have $bp\text{-}cl(A, B) \subseteq bp\text{-}cl(C, D)$ and $bp\text{-}cl(A, B) \subseteq (U, V)$, we have $bp\text{-}cl(C, D) \subseteq (U, V)$, whenever $(C, D) \subseteq (U, V)$ and (U, V) is binary g-open. Hence (C, D) is also a binary $g^{\star}p$-closed subset of M.

Theorem 2.13. *If a subset (A, B) is a binary $g^{\star}p$-closed set if and only if $bp\text{-}cl(A, B) - (A, B)$ contains no nonempty, binary closed set.*

Proof. Necessity. Let (E, F) be binary g-closed set in (X, Y, M), such that $(E, F) \subseteq bp\text{-}cl(A, B) - (A, B)$. Then $(A, B) \subseteq (X, Y) - (E, F)$. Since (A, B) is binary $g^{\star}p$-closed set and $(X, Y) - (E, F)$ is binary g-open then $bp\text{-}cl(A, B) \subseteq (X, Y) - (E, F)$. That is $(E, F) \subseteq (X, Y) - bp\text{-}cl(A, B)$. So $(E, F) \subseteq [(X, Y) - bp\text{-}cl(A, B)] \cap [bp\text{-}cl(A, B) - (A, B)]$. Therefore $(E, F) = (\varphi, \varphi)$.

Sufficiency. Let us assume that $bp\text{-}cl(A, B) - (A, B)$ contains no non empty binary g-closed set. Let $(A, B) \subseteq (U, V)$, (U, V) is binary g-open. Suppose that $bp\text{-}cl(A, B)$ is not contained in (U, V), $bp\text{-}cl(A, B) \cap (U, V)^c$ is non empty, and binary g-closed set of $bp\text{-}cl(A, B) - (A, B)$ which is a contradiction. Therefore $bp\text{-}cl(A, B) \subseteq (U, V)$, and hence (A, B) is binary $g^{\star}p$-closed set.

Theorem 2.14. *If (A, B) is both binary g-open and binary $g^{\star}p$-closed set in (X, Y), then (A, B) is binary g-closed set.*

Proof. . Since (A, B) is binary g-open and binary $g^{\star}p$-closed set in (X, Y), $bp\text{-}cl(A, B) \subseteq (U, V)$. But $(A, B) \subseteq bp\text{-}cl(A, B)$. Therefore $(A, B) = bp\text{-}cl(A, B)$. Since (A, B) is binary closed and $b\text{-}int(A, B) = (A, B)$, this implies $bp\text{-}cl(A, B) = (A, B)$. Hence (A, B) is binary g-closed set.

3. Binary $g^{\star}p$-open set

Definition 3.1. *A subset (A, B) of a binary topological space (X, Y, M) is called binary generalized star pre-open set (briefly, binary $g^{\star}p$-open),if $(A, B)^c$ is binary $g^{\star}p$-closed.*

Theorem 3.2. (1) *If (A, B) is binary open set in (X, Y, M), then it is binary $g^{\star}p$-open.*

(2) *If (A, B) is binary $g^{\star}$-open set in (X, Y, M), then it is binary $g^{\star}p$-open.*

(3) *If (A, B) is binary gp-open set in (X, Y, M), then it is binary $g^{\star}p$-open.*

Proof. It follows from the Theorem 2.2, 2.6 and 2.8.

Remark 3.3. *For subset (A, B) of a binary topological space (X, Y, M),*

(1) $(X, Y) - bg^{\star}p\text{-}int(A, B) = bg^{\star}p\text{-}cl((X, Y) - (A, B))$

(2) $(X, Y) - bg^{\star}p\text{-}cl(A, B) = bg^{\star}p\text{-}int((X, Y) - (A, B))$

Theorem 3.4. *A subset $(A, B) \subseteq (X, Y)$ is binary $g^{\star}p$-open if and only if $(E, F) \subseteq bp\text{-}int(A, B)$ whenever (E, F) is binary g-closed set and $(E, F) \subseteq (A, B)$.*

Proof. Let (A, B) be binary $g^{\star}p$-open set and suppose $(E, F) \subseteq (A, B)$, where (E, F) is binary g-closed. Then $(X, Y) - (A, B)$ is binary $g^{\star}p$-closed set contained in the binary g-open set $(X, Y) - (E, F)$. Hence $bp\text{-}cl((X, Y) - (A, B)) \subseteq (X, Y) - (E, F)$ and $(X, Y) - bp\text{-}int(A, B) \subseteq (X, Y) - (E, F)$. Thus $(E, F) \subseteq bp\text{-}int(A, B)$.

Conversely, if (E, F) is binary g-closed set with $(E, F) \subseteq bp\text{-}int(A, B)$ and $(E, F) \subseteq (A, B)$. Then $(X, Y) - bp\text{-}int(A, B) \subseteq (X, Y) - (E, F)$. Thus $bp\text{-}cl((X, Y) - (A, B)) \subseteq (X, Y) - (E, F)$. Hence $(X, Y) - (A, B)$ is binary $g^{\star}p$-closed set and (A, B) is binary $g^{\star}p$-open set.

Theorem 3.5. *If* $bp\text{-}int(A, B) \subseteq (C, D) \subseteq (A, B)$ *and if* (A, B) *is binary* $g^{\star}p$*-open, then* (C, D) *is binary* $g^{\star}p$*-open.*

Proof. Let $bp\text{-}int(A, B) \subseteq (C, D) \subseteq (A, B)$, then $(A, B)^c \subseteq (C, D)^c \subseteq bp\text{-}cl(A, B)^c$, where $(A, B)^c$ is binary $g^{\star}p$-closed and hence $(C, D)^c$ is also binary $g^{\star}p$-closed by Theorem 2.12. Therefore (C, D) is binary $g^{\star}p$-open.

Remark 3.6. *If* (A, B) *and* (C, D) *are binary* $g^{\star}p$*-open subset of a binary topological space, then* $(A, B) \cup (C, D)$ *is also binary* $g^{\star}p$*-open in* (X, Y)*, as seen from the following example.*

Example 3.7. *In Example 2.11, the binary* $g^{\star}p$*-open sets are* $\{(\varphi, \varphi), (\varphi, \{1\}), (\varphi, \{2\}), (\varphi, Y), (\{a\}, \{1\}), (\{a\}, Y), (\{b\}, \{2\}), (\{b\}, Y), (X, Y)\}$. *Let* $A = (\varphi, \{1\})$ *and* $B = (\{b\}, \{2\})$ *are binary* $g^{\star}p$*-open sets. Then* $A \cup B = (\varphi, \{1\}) \cup (\{b\}, \{2\}) = (\{b\}, Y)$ *is also binary* $g^{\star}p$*-open set.*

4. Binary $g^{\star}p$-interior and binary $g^{\star}p$-closure

Definition 4.1. *Let* (X, Y, M) *be a binary topological space and let* $(x, y) \in (X, Y)$*. A subset* (U, V) *of* (X, Y) *is said to be binary* $g^{\star}p$*-neighbourhood of* (x, y) *if there exists an binary* $g^{\star}p$*-open set* (G, H) *such that* $(x, y) \in (G, H) \subseteq (U, V)$*.*

Definition 4.2. (1) $bg^{\star}p\text{-}int(A, B) = \bigcup \{(C, D) : (C, D)$ *is binary* $g^{\star}p$*-open set and* $(C, D) \subseteq (A, B)\}$

(2) $bg^{\star}p\text{-}cl(A, B) = \bigcap\{(C, D) : (C, D)$ *is binary* $g^{\star}p$*-closed set and* $(A, B) \subseteq (C, D)\}$

Theorem 4.3. *If* (A, B) *be a subset of* (X, Y). *Then* $bg^{\star}p\text{-}int(A, B) = \bigcup\{(C, D) : (C, D)$ *is binary* $g^{\star}p$*-open set and* $(C, D) \subseteq (A, B)\}$.

Proof. Let (A, B) be a subset of (X, Y). $(x, y) \in bg^{\star}p\text{-}int(A, B)$.

$\Leftrightarrow (x, y)$ is a binary $g^{\star}p$-interior point of (A, B).

$\Leftrightarrow (A, B)$ is a binary $g^{\star}p$-neighbourhood of point (x, y).

$\Leftrightarrow$ There exists binary $g^{\star}p$-open set (C, D) such that $(x, y) \in (C, D) \subseteq (A, B)$.

$\Leftrightarrow (x, y) \in \bigcup\{(C, D) : (C, D)$ is binary $g^{\star}p$-open set and $(C, D) \subseteq (A, B)\}$

Hence, $bg^{\star}p\text{-}int(A, B) = \bigcup\{(C, D) : (C, D)$ is binary $g^{\star}p$-open set and $(C, D) \subseteq (A, B)\}$.

Theorem 4.4. *Let* (A, B) *and* (C, D) *be subsets of* (X, Y). *Then*

(1) $bg^{\star}p\text{-}int(X, Y) = (X, Y)$ *and* $bg^{\star}p\text{-}int(\varphi, \varphi) = (\varphi, \varphi)$

(2) $bg^{\star}p\text{-}int(A, B) \subseteq (A, B)$

(3) *If* (C, D) *is any binary* $g^{\star}p$*-open sets contained in* (A, B), *then* $(C, D) \subseteq bg^{\star}p\text{-}int(A, B)$

(4) *If* $(A, B) \subseteq (C, D)$, *then* $bg^{\star}p\text{-}int(A, B) \subseteq bg^{\star}p\text{-}int(C, D)$

(5) $bg^{\star}p\text{-}int(bg^{\star}p\text{-}int(A, B)) = bg^{\star}p\text{-}int(A, B)$.

Proof. (1) Since (X, Y) and (φ, φ) are binary $g^{\star}p$-open sets, by Theorem 4.3 $bg^{\star}p\text{-}int(X, Y) = \bigcup\{(C, D) : (C, D)$ is $bg^{\star}p$-open and $(G, H) \subseteq (X, Y)\}$ $= (X, Y) \bigcup\{(A, B) : (A, B)$ is a binary $g^{\star}p$-open set$\} = (X, Y)$. Since, (φ, φ) is the only binary $g^{\star}p$-open set contained in (φ, φ), $bg^{\star}p\text{-}int(\varphi, \varphi) = (\varphi, \varphi)$.

(2) Let $(x, y) \in bg^{\star}p\text{-}int(A, B) \Rightarrow (x, y)$ is a binary $g^{\star}p$-interior point of (A, B).

$\Rightarrow (A, B)$ is a binary $g^{\star}p$-neighbourhood of (x, y).

$\Rightarrow (x, y) \in (A, B)$. Thus, $(x, y) \in bg^{\star}p\text{-}int(A, B) \subseteq (A, B)$.

(3) Let (C, D) be any binary $g^{\star}p$-open set such that $(C, D) \subseteq (A, B)$. Let $(U, V) \in (C, D)$, then, (C, D) is a binary $g^{\star}p$-open set contained (x, y) in (A, B) is a binary $g^{\star}p$-interior point of (A, B). That is (C, D) is a $bg^{\star}p\text{-}int(A, B)$. Hence $(C, D) \subseteq bg^{\star}p\text{-}int(A, B)$.

(4) Let (A, B) and (C, D) be subsets of (X, Y) such that $(A, B) \subseteq (C, D)$. Let $(x, y) \in bg^{\star}p\text{-}int(A, B)$. Then (x, y) is a binary $g^{\star}p$-interior point of (A, B) and so (A, B) is binary $g^{\star}p$-neighbourhood of (x, y). This implies that $(x, y) \in bg^{\star}p\text{-}int(C, D)$. Thus we have shown that $(U, V) \in bg^{\star}p\text{-}int(C, D)$. Hence, $bg^{\star}p\text{-}int(A, B) \subseteq bg^{\star}p\text{-}int(C, D)$.

(5) Let (A, B) be any subset of (X, Y). Then by definition of binary $g^{\star}p$-interior, $bg^{\star}p\text{-}int(A, B) = \bigcap\{(A, B) \subseteq (E, F) \in bg^{\star}p\text{-}cl(X, Y)\}$ if $(A, B) \subseteq (E, F) \in bg^{\star}p\text{-}cl(X, Y)$, then $bg^{\star}p\text{-}int(A, B) \subseteq (E, F)$. Since (E, F) is a binary $g^{\star}p$-closed set containing $bg^{\star}p\text{-}int(A, B)$. By (3), $bg^{\star}p\text{-}int(bg^{\star}p\text{-}int(A, B)) \subseteq (E, F)$. Hence $bg^{\star}p\text{-}int(bg^{\star}p\text{-}int(A, B)) \subseteq \bigcap\{(A, B) \subseteq (E, F) \in bg^{\star}p\text{-}cl(X, Y)\} = bg^{\star}p\text{-}cl(A, B)$. That is, $bg^{\star}p\text{-}int(bg^{\star}p\text{-}int(A, B)) = bg^{\star}p\text{-}int(A, B)$.

Theorem 4.5. *If a subset (A, B) of a space (X, Y) is binary $g^{\star}p$-open then $bg^{\star}p\text{-}int(A, B) = (A, B)$.*

Proof. Let (A, B) be a binary $g^{\star}p$-open subset of (X, Y). We know that $bg^{\star}p\text{-}int(A, B) \subseteq (A, B)$. Also (A, B) is binary $g^{\star}p$-open set contained in (A, B). From Theorem 4.4(3), $(A, B) \subseteq bg^{\star}p\text{-}int(A, B)$. Hence, $bg^{\star}p\text{-}int(A, B) = (A, B)$.

Theorem 4.6. *If (A, B) and (C, D) are subsets of (X, Y), then $bg^{\star}p\text{-}int(A, B) \cup bg^{\star}p\text{-}int(C, D) \subseteq bg^{\star}p\text{-}int((A, B) \cup (C, D))$.*

Proof. We know that $(A, B) \subseteq (A, B) \cup (C, D)$ and $(C, D) \subseteq (A, B) \cup (C, D)$ and we have by Theorem 4.4(4), $bg^{\star}p\text{-}int(A, B) \subseteq bg^{\star}p\text{-}int((A, B) \cup (C, D))$ and $bg^{\star}p\text{-}int(C, D) \subseteq bg^{\star}p\text{-}int((A, B) \cup (C, D))$. This implies that $bg^{\star}p\text{-}int(A, B) \cup bg^{\star}p\text{-}int(C, D) \subseteq bg^{\star}p\text{-}int((A, B) \cup (C, D))$.

Theorem 4.7. *If (A, B) and (C, D) are subsets of space (X, Y), Then $bg^{\star}p\text{-}int((A, B) \cap (C, D)) = (bg^{\star}p\text{-}int(A, B) \cap bg^{\star}p\text{-}int(C, D))$*

Proof. We know that $(A, B) \cap (C, D) \subseteq (A, B)$ and $(A, B) \cap (C, D) \subseteq (C, D)$. We have, by Theorem 4.4(4), $bg^{\star}p\text{-}int((A, B) \cap (C, D)) \subseteq bg^{\star}p\text{-}int(A, B)$ and $bg^{\star}p\text{-}int((A, B) \cap (C, D)) \subseteq bg^{\star}p\text{-}int(C, D)$. This implies that

$$bg^{\star}p\text{-}int((A, B) \cap (C, D)) \subseteq bg^{\star}p\text{-}int(A, B) \cap bg^{\star}p\text{-}int(C, D) \tag{1}$$

Again, Let $(x, y) \in bg^{\star}p\text{-}int(A, B) \cap bg^{\star}p\text{-}int(C, D)$. Then $(x, y) \in bg^{\star}p\text{-}int(A, B)$ and $(x, y) \in bg^{\star}p\text{-}int(C, D)$. Hence, (x, y) is a binary $g^{\star}p$-interior point of each sets (A, B) and (C, D) is binary $g^{\star}p$-neighbourhood of (x, y), So that their intersection $(A, B) \cap (C, D)$ is also binary $g^{\star}p$-neighbourhood of (x, y) hence $(x, y) \in bg^{\star}p\text{-}int((A, B) \cap (C, D))$. Therefore

$$bg^{\star}p\text{-}int(A, B) \cap bg^{\star}p\text{-}int(C, D) \subseteq bg^{\star}p\text{-}int((A, B) \cap (C, D)). \tag{2}$$

From 1 & 2, we get $bg^{\star}p\text{-}int((A, B) \cap (C, D)) = bg^{\star}p\text{-}int(A, B) \cap bg^{\star}p\text{-}int(C, D)$.

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