# Study of Induction Motor Dynamics Through Simulation on MATLAB Simulink

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#### **Abstract**

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The dynamics and transient characteristics of single phase or three phase induction motor has always been a challenging field for researchers. This is because of the natural interaction established between the motor parameters in stator and rotor and various parameters of the system employing the motor. This mutual interaction results in nonlinear differential equations of motion. With the help of a-b-c three phase to d-q axis two phase transformations; it has been possible to write comparatively simpler but still nonlinear dynamic equations of induction motor to deduce its transient and steady state behavior. This concept has been well discussed in various literatures. This paper reviews the progress made in the field of various application based computational approach used for the dynamic analysis of induction motor. With the help of flux model simulation of induction motor on MATLAB/Simulink current and torque behavior has been studied closely. Attempts are made to highlight the electrical and mechanical losses the induction motor imposes on the entire system for future research.

## Introduction

Induction motors (IM) are the most common motors used in industrial motion control systems Although AC induction motors are easier to design than DC motors, the speed and torque control in various types of induction motors require a greater understanding of the design and the characteristics of the motors.

Because of the absence of back EMF at the start, IM draws high inrush current causing voltage dip in supply line and heating loss. In variable speed drive IM normally constitutes an element within a feedback loop so its transient behavior must be considered to design any control circuit. Subsequently, the high performance drive control is generally based on the dynamic model of IM.

#### Literature Review

The transient modeling of induction machines continues to receive colossal deliberation as the transient behavior of induction motor has imperative effect on the overall performance of the whole system. In variable speed drive, IM is usually converter fed. Consequently, the dynamics of IM largely affect the performance of converter and the filter. The prerequisite of the appropriate mathematical model and computation techniques increase the complexity of dynamic modeling. Though, through the generalized dq axis concept and space vector theory the dynamic model of most of the electrical machine has been discussed in many books and research papers [5], [6], [9], [10].

ISSN NO: 2249-3034

In [13] the analysis of the three phase squirrel cage induction motor was different. The coupling impedances method was used; the motor was represented as a set of equations in a matrix form to be solved to give stator, rotor loop and end-ring currents. The generalized harmonic analysis using the coupling impedance method was used to predict the performance of cage rotor induction motor with stator faults in [14]. The coupling impedance method is not valid for the analysis of the cage motors with dynamic load conditions such as a reciprocating compressor, because the load is time-varying and this method is only valid for steady state condition.

The finite element method (FEM) was used to analyze the wound rotor induction motor transient torque and current in [15]. This method used the circuit equation approach and coupled magnetic field. It can be extended to accommodate a cage rotor induction motor relatively easily. The FEM approach is valid for dynamic load analysis, but it is relatively expensive in terms of computational time.

In [16, 17] the dynamic model of a cage rotor induction motor was proposed based on the winding function approach and coupled magnetic circuit approach. The model is a set of differential equations for m-phase induction machine with n rotor bars. The parameters of the model are assumed to be calculated directly from the geometry and winding layout of the machine using the provided equations. This model was used to predict the behavior of multiphase induction machine during the transient as well as steady state behavior including

the effects of stator asymmetry, broken rotor bars, and broken end-rings.

A d-q model for the dynamic behavior of an induction motor including the effects of saturation on leakage and magnetizing inductance was introduced in [18]. The motor is represented by a set of differential equations in the d-q reference frame. The stator leakage inductance was replaced by the sum of the air-dependent end-winding leakage inductance and the iron-dependent saturated leakage inductance. The same thing applies to the rotor winding in the case of a wound rotor machine. The model parameters such as stator and rotor resistances and reactance were measured using no-load test and locked-rotor test.

A modified winding function approach (MWFA) has been used in [19] to analyze the dynamic performance of a single phase squirrel cage induction motor with a eccentric rotor. The motor was mathematically modeled by a set of differential equations in a matrix form that would be solved by a numerical method to get the currents in the stator and rotor loops. The model was based on the coupled magnetic circuit technique. The stator circuit has three parts; two main windings and an auxiliary starting winding. In the computation of the stator windings voltages therefore (two main windings and auxiliary winding), the voltage across the capacitor should be included. The same approach was used to analyze the performance of a three phase induction motor under mixed eccentricity conditions in [20], finite element method was used to substantiate the inductance values.

A modified two-axis mathematical model for a three phase induction motor has been developed to include the skin effect, the temperature influence on the parameters and allowing for the stator and rotor windings and stator and rotor core average temperature evaluation [21]. This model is useful for any dynamic studies mainly those including fast motor speed changes and intermittent loads. In this paper the model parameters were either measured using the no-load and locked-rotor methods or provided by the motor manufacturers. There are no equations provided to compute the inductances.

Space magnetic motive force (MMF) harmonics are often ignored to make the model simpler; however, in the proposed model of [22] that is based on the rotating field theory, the mmf harmonics are not ignored. Simple connection matrices are used to model winding interconnections. This unified approach was

proposed to analyze steady-state performance of single phase cage motor having any number of windings interconnected in any conceivable manner, with any number of capacitors. This approach has used the coupling impedances technique to build the model of the single phase induction motor, but the coupling impedances technique is valid only for steady state analysis.

A theoretical analysis model of induction motor was made based on rotating field theory in [23] for a wound rotor and squirrel cage induction motor to study the behavior of these motors in a healthy case and two faulty cases. The faulty cases included a single broken bar fault and an imbalanced stator voltage supply fault. The model was based on the machine variables (a-b-c), but the torque equation used the d-q-axis currents components for the stator and rotor which required an additional transformation step. A Simulink software package was used to model the motor and plot the results. The load applied to the motor was steady.

A mathematical dynamic model for a three phase induction motor which is able to reproduce the induction motor electromagnetic torque oscillations at steady state due to air-gap irregularities was described in [24]. In this paper, equations for the self and mutual winding inductances have been derived. This approach basically modifies the inductance matrices elements to include the irregularities of the stator and rotor. The derivation of this model was based on a wound-rotor induction machine that has an elliptical rotor surface form.

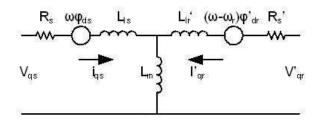
Another type of dynamic model used for induction motors was based on the modified two-axes equivalent circuit of the motor [25]. 3D and 2D finite element techniques were used to determine the parameters of those equivalent circuits. This approach is quite expensive from the computational time point of view. It cannot predict the rotor bar currents in the case of squirrel cage induction motor.

In [26] a generalized two-ax (d-q-axes) model for a squirrel cage induction motor was developed based on the winding function approach and coupled magnetic circuit theory. In this paper the loop inductances for healthy case or broken bars case was done by the winding function approach on the machine variables (a-b-c) to build the rotor inductances matrix, then transformed into equivalent dq matrix. In this approach many transformation processes were needed, because the inductance matrix depends on the rotor angle

position which should be updated each calculation step of the simulation. The results of the simulation were the dq components of the rotor currents but did not have information on each physical rotor bar current.

A mathematical dynamic model based on a d-q transformation has been used in [28] to simulate the steady-state performance of squirrel cage induction motor. The parameters of the motor were measured by the two well-known tests; no-load test and locked-rotor test. The load applied to the motor was a steady load; the only dynamic analysis of the motor was the transient response of the motor during starting. This model like any other dq-model cannot predict the current in each rotor loop or bar.

#### **Motor Dynamic Modeling**



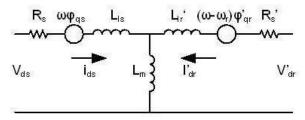


Fig. 1. d-q model of three phase induction motor

The induction motor is modeled as an ideal cylindricalrotor machine with uniform air gap and stator windings neglecting the skin effect, saturation, eddy current and temperature effect. The differential governing the transient performance of the induction motor can be described in several ways and they only differ in detail and in their suitability for use in a given application. If the speed of the motor is assumed constant, then the motor differential equations become linear and analytical method can be used to obtain the motor torque and currents. However, where changes of motor speed have to be accounted for, analytical highly inadequate method becomes differential equations are non-linear and could only be solved numerically using digital or analog computers. The d-q axis model of the motor

provides a convenient way of modeling the machine and is most suitable for numerical solution. This is preferable to the space-vector motor model which describes the motor in terms of complex variables. The d-q equivalent circuit is shown in Fig. 1.

From Fig.1 the differential equations describing the dynamic performance of the motor in arbitrary reference frame are given as in [6] with all the rotor parameters referred to the stator. The prime on the referred values have been omitted convenience.

$$\begin{bmatrix} V_{qs} \\ V_{ds} \\ V_{qr} \\ V_{ds} \end{bmatrix} = \begin{bmatrix} R_S + \mathbf{I}_{sl} \mathcal{D} & & & & & & & & & \\ -\omega \mathbf{I}_{S} & R_S + \mathbf{I}_{S} \mathcal{D} & -\omega \mathbf{I}_{m} & \mathbf{I}_{sm} \mathcal{D} \\ & -\omega \mathbf{I}_{S} & R_S + \mathbf{I}_{S} \mathcal{D} & -\omega \mathbf{I}_{m} & \mathbf{I}_{sm} \mathcal{D} \\ & \mathbf{I}_{sm} \mathcal{D} & & & & & & & \\ & \mathbf{I}_{cm} \mathcal{D} & & & & & & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & R_r + \mathbf{I}_{r} \mathcal{D} & & & & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & R_r + \mathbf{I}_{r} \mathcal{D} & & & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & R_r + \mathbf{I}_{r} \mathcal{D} & & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & R_r + \mathbf{I}_{r} \mathcal{D} & & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & R_r + \mathbf{I}_{r} \mathcal{D} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & R_r + \mathbf{I}_{r} \mathcal{D} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & R_r + \mathbf{I}_{r} \mathcal{D} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & R_r + \mathbf{I}_{r} \mathcal{D} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & R_r + \mathbf{I}_{r} \mathcal{D} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & R_r + \mathbf{I}_{r} \mathcal{D} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & R_r + \mathbf{I}_{r} \mathcal{D} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & R_r + \mathbf{I}_{r} \mathcal{D} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & R_r + \mathbf{I}_{r} \mathcal{D} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & R_r + \mathbf{I}_{r} \mathcal{D} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & & \\ & -(\omega \mathbf{I}_{cm} - \omega \mathbf{I}_{cm}) \mathbf{I}_{m} & & \\ & -(\omega \mathbf{I}_$$

$$L_s = L_{ls} + L_{m} \tag{2}$$

$$L_r = L_{br} + L_{m} \tag{3}$$

$$p = \frac{d}{dt} \tag{4}$$

The equation used for the prediction of electromagnetic

torque is given by [3],  

$$T_e = \frac{3}{2} \frac{P}{2} L_{ex} \left( i_{qe} i_{dr} - i_{dz} i_{qe} \right)$$
(5)

And the rotor speed is given as
$$\frac{d\omega_r}{dt} = \frac{P}{2I} (T_e - T_L)$$
(6)

Where T<sub>e</sub> is the electromagnetic torque and T<sub>L</sub> is the load torque, P is the number of poles

The relationship between the actual3-phase voltages Vas, Vbs and Vcs and the d-q axis voltages of equation

$$\begin{bmatrix} V_{qs} \\ V_{ds} \\ V_{qr} \\ V_{dr} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} V_{as} \\ V_{bs} \\ V_{cs} \end{bmatrix}$$

(7)

#### **Steady State Analysis**

The steady state mathematical model is obtained by equating all derivative terms in equation (1) to zero and with the machine described in synchronously rotating reference frame ( $\omega = \omega_e$ ) as specified in equation (8).

$$\begin{bmatrix} \mathbf{0}_{n0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{n} & \omega_{n}\mathbf{I}_{n} & \mathbf{0} & \omega_{n}\mathbf{I}_{m} \\ -\omega\mathbf{I}_{n} & \mathbf{R}_{n} & -\omega_{n}\mathbf{I}_{m} & \mathbf{0} \\ \mathbf{0} & (\omega - \omega_{n})\mathbf{I}_{m} & \mathbf{R}_{n} & (\omega_{n} - \omega_{n})\mathbf{I}_{m} \\ -(\omega_{n} - \omega_{n})\mathbf{I}_{m} & \mathbf{0} & -(\omega_{n} - \omega_{n})\mathbf{I}_{n} & (\mathbf{0})\mathbf{R}_{n} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{qn0} \\ \mathbf{I}_{qn0} \\ \mathbf{I}_{qn0} \\ \mathbf{I}_{qn0} \end{bmatrix}$$

Simultaneously, the torque is given as

$$T_{eo} = \frac{3P}{2} L_m (i_{qso} i_{dro} - i_{dso} i_{qso})$$
(9)

Where,  $i_{qso}$ ,  $i_{dso}$ ,  $i_{qro}$  and  $i_{dro}$  are the steady-state

currents and  $\omega_e$ , is the motor synchronous speed.

#### **Transient Analysis**

The differential equations describing the transient behavior of the motor in stationary reference frame are obtained by equating in equation (3.1) to zero. Therefore.

$$\begin{bmatrix} V_{ds} \\ V_{ds} \\ V_{dr} \\ V_{dr} \end{bmatrix} = \begin{bmatrix} R_s + L_s p & 0 & L_{mp} & 0 \\ 0 & R_s + L_s p & 0 & L_{mp} \\ L_{mp} & -\omega_r L_m & R_r + L_r p & \omega_r L_{mp} \\ \omega_r L_m & L_{mp} & \omega_r L_r & R_s + E_r p \end{bmatrix} \begin{bmatrix} \bar{t}_{qq} \\ \bar{t}_{ds} \\ \bar{t}_{qr} \\ \bar{t}_{ds} \end{bmatrix}$$
(10)

Equation (11) can be put in matrix from as follows:

$$[V] = [L]P[i] + \omega_r[G] + [R][i]$$

(11)

Where,

$$[V] = [V_{qs} \quad V_{ds} \quad 0 \quad 0]$$
 (12)

$$[R] = \begin{bmatrix} R_s & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 \\ 0 & 0 & R_s & 0 \\ 0 & 0 & 0 & R_s \end{bmatrix}$$
(13)

$$[L] = \begin{bmatrix} L_s & 0 & L_m & 0 \\ 0 & L_s & 0 & L_m \\ L_m & 0 & L_r & 0 \\ 0 & L_m & 0 & L_r \end{bmatrix} ...$$
(14)

$$[G] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -L_m & 0 & -L_r \\ L_m & 0 & L_r & 0 \end{bmatrix} \dots$$
(15)

$$[i] = [i_{qs} \quad i_{ds} \quad i_{qr} \quad i_{dr}]^{\dagger}$$
(16)

### d-q modeling of Induction Motor

The differential equations to comprehensively study the steady-state and dynamic performance of induction machines were developed in the machine variables using Kirchhoff's law and Newton law of motion. That is, the motor steady-state and transient behavior are described using the a-b-c stator and rotor quantities. The d-q-0 quantities reduce the complexity of the resulting differential equations that maps the dynamics of induction machines. The three phase motor variables are transformed to d-q-0 axis using Park's transformations.

The three phase voltages are first converted into two phase stationary reference frame  $(\alpha-\beta)$  and then the frame variables are transformed to the synchronously rotating reference frame (d-q). The relationship between these reference frames is given in matrices expressions (21) and (22). Here  $\theta$  is the transformation angle.

$$\begin{bmatrix} V_{\alpha} \\ V_{\beta} \end{bmatrix} = \frac{\sqrt{2}}{3} \begin{vmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{vmatrix} \begin{bmatrix} V_{\alpha} \\ V_{\beta} \\ V_{\alpha} \end{bmatrix}$$
(21)

$$\begin{bmatrix} V_d \\ V_g \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} V_{\alpha} \\ V_{\beta} \end{bmatrix}$$
 (22)

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \frac{\sqrt{2}}{2} \begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} V_0 \\ V_b \\ V_c \end{bmatrix} \quad (23)$$

The d-q axis variables are again transformed to three phase variable by means of the relationship mentioned from expressions (24) to (26).

$$\begin{bmatrix} I_{\alpha} \\ I_{\beta} \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} I_{d} \\ I_{\alpha} \end{bmatrix}$$
 (24)

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{\sqrt{2}}{3} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} I_{\alpha} \\ I_{\beta} \end{bmatrix}$$
 (25)

$$\begin{bmatrix}
I_a \\
I_b \\
I_e
\end{bmatrix} = \frac{\sqrt{2}}{3} \begin{bmatrix}
1 & 0 \\
-\frac{1}{2} & \frac{\sqrt{3}}{2} \\
-\frac{1}{3} & -\frac{\sqrt{3}}{2}
\end{bmatrix} \begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{bmatrix} \begin{bmatrix}
I_d \\
I_q
\end{bmatrix}$$
(26)

# **Dynamic Modeling using State Space Model**

The dynamic machine model in state space form is important for transient analysis simulation. Though the synchronously rotating frame model is normally preferred, the stationary frame models are also used.

#### **Synchronously Rotating Reference Frame**

For a two phase machine as shown in Fig.1, we need stator variables ( $d^s$ - $q^s$ ) and rotor variables ( $d^r$ - $q^r$ ) are written in synchronously rotating reference frame  $d^e$ - $q^e$ . The stator circuit equations are given in equations (27) to (30) and rotor circuit is represented by equations (31) to (34) [2],[3],[4],[5].

$$v_{qs}^s = R_{si_{qs}^s} + \frac{d}{dt} \varphi_{qs}^s \tag{27}$$

$$v_{ds}^{s} = R_{si_{ds}^{s}} + \frac{d}{ds} \varphi_{ds}^{s}$$
 (28)

Here,  $\varphi_{qq}^{\bullet}$  and  $\varphi_{dq}^{\bullet}$  are the q-axis and d-axis stator flux linkages respectively. When these equations are converted to  $d^{e}$ -  $q^{e}$  frame, the following equations can be written:

$$v_{qs} = R_s i_{qs} + \frac{d}{dt} \varphi_{qs} + \omega_s \varphi_{ds}$$
 (29)

$$v_{ds} = R_s i_{ds} + \frac{d}{dt} \varphi_{ds} - \omega_e \varphi_{qs} \qquad (30)$$

Where, all the variables are in rotating frame. The last term in equation (29) and (30) can be defined as speed emf due to rotation of axis.

If the rotor is not moving, i.e.,  $\omega_r = 0$ , the rotor equations are:

$$v_{qr} = R_r i_{qr} + \frac{d}{dt} \varphi_{qr} + \omega_{\epsilon} \varphi_{dr} \quad (31)$$

$$v_{dr} = R_r i_{dr} + \frac{d}{dt} \varphi_{dr} - \omega_e \varphi_{qr}$$
 (32)

Where, all the variables and parameters are referred to stator. Since the rotor actually moves at speed  $\omega_r$ , the dq axis fixed on the rotor move at a speed of  $\omega_e$  -  $\omega_r$  relative to synchronously rotating frame.

Therefore, de-qe frame, the rotor equations should be modified as:

$$v_{qr} = R_r i_{qr} + \frac{d}{dt} \varphi_{qr} + (\omega_e - \omega_r) \varphi_{dr}$$
33)

$$v_{dr} = R_r i_{dr} + \frac{d}{dt} \varphi_{dr} + (\omega_e - \omega_r) \varphi_{qr}$$
(34)

From Fig. 1 the flux linkage expressions in terms of current are given in equation (35) to (40).

$$\varphi_{as} = L_{ls}i_{as} + L_m(i_{as} + i_{ar}) \qquad (35)$$

$$\varphi_{qs} = L_{lr}i_{qr} + L_m(i_{qs} + i_{qr}) \qquad (36)$$

$$\varphi_{qm} = L_m(i_{qs} + i_{qr}) \tag{37}$$

$$\varphi_{ds} = L_{ls}i_{ds} + L_m(i_{ds} + i_{dr}) \tag{38}$$

$$\varphi_{dr} = L_{lr}i_{dr} + L_m(i_{ds} + i_{dr}) \qquad (39)$$

$$\varphi_{dm} = L_m(i_{ds} + i_{dr}) \tag{40}$$

All the expression from (27) to (40) can be manipulated and combined in the matrix as below

$$\begin{bmatrix} V_{qx} \\ V_{dx} \\ V_{qr} \\ V_{gr} \end{bmatrix} = \begin{bmatrix} R_x + SL_x & \omega_x L_x & SL_m & \omega_x L_m \\ -\omega_x L_x & R_x + SL_x & -\omega_x L_m & SL_m \\ SL_m & (\omega_x - \omega_r)L_m & R_r + SL_r & (\omega_x - \omega_r)L_m \\ -(\omega_x - \omega_r)L_m & SL_m & -(\omega_x - \omega_r)L_r & R_r + SL_r \end{bmatrix} \begin{bmatrix} I_{qx} \\ I_{dx} \\ I_{qx} \end{bmatrix}$$

$$(41)$$

Here S is the Laplace operator. The speed  $\omega_r$  in equations (31) and (32) is normally not treated as constant but it is related to the torque.

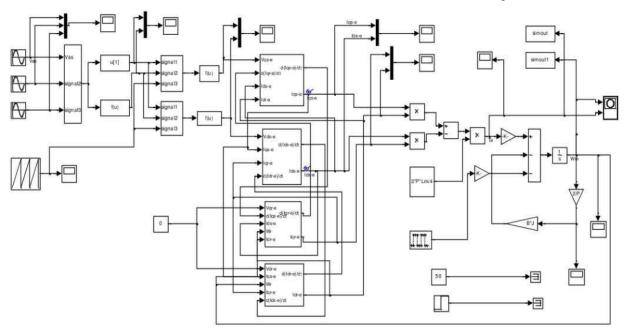


Fig 2 SIMULINK simulation of de-qe flux linkage model of induction motor

$$F_{dr} = \omega_b \varphi_{dr} \qquad (46)$$

Here  $\omega_b$  is the base speed. With the help of expressions from (27) to (46), the state space model of IM in d-q axis is obtained.

$$\frac{dF_{qs}}{dt} = \omega_b \left[ v_{qg} - \frac{\omega_e}{\omega_b} F_{dg} - \frac{R_s}{X_{ls}} (F_{qg} - F_{qqq}) \right] \quad (47)$$

$$\frac{dF_{ds}}{dt} = \omega_b \left[ v_{ds} - \frac{\omega_e}{\omega_b} F_{qs} - \frac{R_s}{\chi_{ls}} (F_{ds} - F_{dm}) \right]$$
(48)

$$\frac{dF_{qr}}{dt} = -\omega_b \left[ \frac{\omega_e - \omega_r}{\omega_b} F_{dr} + \frac{R_r}{\chi_{lr}} (F_{qr} - F_{qm}) \right] \quad (49)$$

$$\frac{dF_{dr}}{dt} = -\omega_b \left[ -\frac{\omega_{e^-} \omega_r}{\omega_h} F_{qr} + \frac{R_r}{K_{tr}} (F_{dr} - F_{dm}) \right] \quad (50)$$

Considering motor electrical torque T<sub>e</sub> as the output the output equation in terms of state variables is

$$T_e = \frac{3}{2} \left( \frac{p}{2} \right) \frac{1}{\omega_b} \left( i_{qs} F_{ds} - i_{ds} F_{qs} \right)$$
 (51)

#### **Dynamic modeling of Induction Motor in Simulink**

Fig. 2 shows the flux model simulation of the de-qe model of the induction motor, which was discussed in previous section. The model receives the input voltages  $v_{qs}$  and  $v_{ds}$  and frequency  $\omega_e$  and solves the output currents  $i_{qs}$  and  $i_{ds}$  using flux linkage equations. At the input, the inverter output voltages  $v_{an}$ ,  $v_{bn}$  and  $v_{cn}$  are converted into  $v_{qs}$  and  $v_{ds}$ . In order to illustrate the method of solution discussed above, results of the performance studies of an induction motor driving a pump load are presented.

The simulations have been carried out using the below motor data obtained from the open and short circuit tests of the induction motor under study. 40hp, 460V, 4Poles, 50Hz, Rs = 0.087, L1s=0.0425H, L1r= O..043H, Lm =0.04H, Rr = 0.187 $\Omega$ , J = 1.662 Kgm<sup>2</sup>, B = 0.01 Nms

The unmasked views of the subsystems in Fig. 2 are given in fFig.3, fig. 4, fig. 5 and fig. 6. The state space model given by equations (47) to (50) has been simulated in these subsystems only to obtain the d-q axis stator and rotor currents. Various results of the simulation are given in Fig. 7 to Fig.12. For the balanced three phase voltages the d-q axis voltage obtained (Fig. 7) shows  $v_d = 0$ . This does not make the direct axis stator and rotor currents zero. Fig.8 and Fig. 9 shows very high inrush d-q axis currents through stator and rotor circuits. These currents settle to their steady state with the motor speed reaches to steady state. The motor torque and speed are more influenced by the quadrature axis stator and rotor current as compared to the direct axis currents. The stator and rotor circuit d-q axis currents are 180° phase apart.

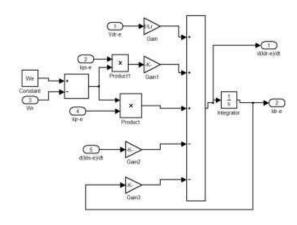


Fig 3 Subsystem

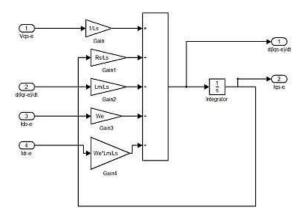


Fig. 4. Subsystem1

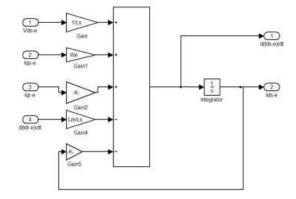


Fig. 5 Subsystem2

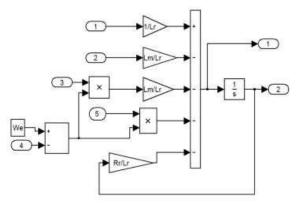


Fig. 6 Subsystem3

After passing through the high magnitude transient, the d-q-axis stator and rotor currents faces second phase of transient. Before going to the second phase of transient q-axis initial sinusoidal under damped response indicating the dominant imaginary poles of the system. This is quite a second order system response nurturing a zero steady state response. After very fast initial transient with high overshoot the IM passes through next phase of transient which is almost oscillatory. There is no damping and system oscillates with its natural frequency. For a second order R-L circuit this phenomenon shows either the presence of a capacitive element or interaction between two inductive elements. The oscillation dies very fast showing some of the system now dominating are very close to origin. The third phase of transient starts now showing the IM a second order system with damping factor near 0.7 -0.8. The q-Axis current reaches to its steady state value. The steady state responses are oscillatory. The motor inherently acts as an oscillator where the natural response is not dying out completely. This is due to the interaction of circuit parameter. This does not lead to the unstably of the system but results in to humming

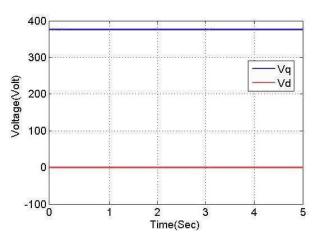


Fig. 7 The d-q Axis voltage of the IM

sound and mechanical vibration.

The d- axis currents in stator and rotor circuits also pass through the same phases of transient as that of q-axis circuits but finally critically responded like a first order system. This implicates dominating real poles of the induction motor. With positive gain the rotor d-axis

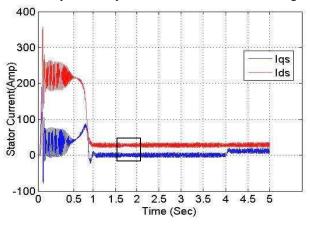


Fig. 8. d-q axis stator current

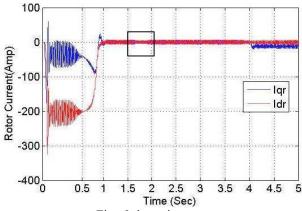
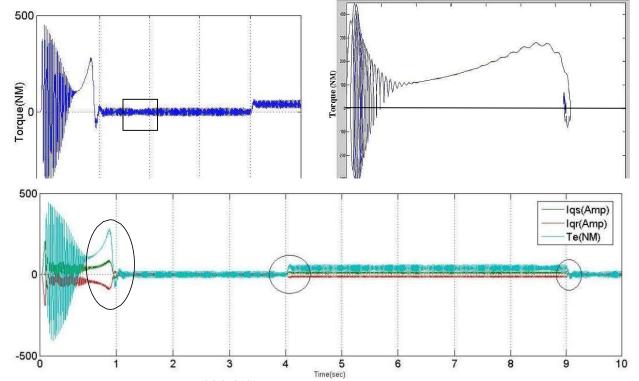


Fig. .9 d-q axis rotor current



Volume XII.I, Is, sue II., FEBRUARY/2024 Fig. 13 Perio dic change and os cill ation in q-axis currents and torque

current settled around zero whereas the stator d-axis current has negative gain and the positive steady state current. This means that the q-axis equivalent circuits of the induction motor are more interactive and nonlinear as compared to the d-axis circuits.

There are another transient observed in the q- axis stator and rotor current and also in the torque which is periodic as shown encircled in Fig 13. Observation with increased simulation time shows that the q-axis currents and the output torque waveforms are rectangular with the oscillation imposed on it This also results in periodic spikes in currents and torque. This behavior of induction motor does not allow the motor speed to remain constant as the torque never goes to zero following the ideal torque speed characteristic of the induction motor. This loss in the rotor torque results in heating loss in long run and may result in shaft and bearing damage. Apart from this periodic thinning of waveform has also been observed everywhere marked with square box. Induction motor is a sinking unit where large amount of energy get dissipated.

The dynamics of induction motor will be more interesting when it is inverter fed.

#### Conclusion

The paper presents a simple method of analyzing and simulating the steady state and transient state performances of an induction motor based on already published books and research papers. The simulation results presented in this paper will indeed provide essential information about the motor performances prior to its design.

The MATLAB/Simulink based model has been made using the simple subsystems based on the state space flux model of the induction motor. The model is quite reliable and predicts the expected results. The simulation model and results are useful for the designers of induction motor as the performance of the motor can be predicted by varying its parameters and inertia for dynamic load change

The induction motor model developed can be in an advanced drive system, eg. Field oriented control. The model can be used to analyze any type of induction motor regardless of the stator winding arrangement for any dynamic load.

This analysis is very helpful for squirrel cage induction motor connected to reciprocating pumps or compressor, especially when the motor is chosen to be on the minimum required rating for normal operation without considering the pulsating torques. In such a case, the motor will stall at the first pulsating torque that requires more torque than maximum ability of the motor. The choice is either a higher rated motor that will be good

enough to overcome this difficulty for a few seconds and then there will be no need for its higher rated power for the rest of the operation, or to keep the existing motor that has the minimum required rating to do the normal operation, and it can overcome the pulsating torque by the aid of using fly wheel. pulsating to torque by the aid of using fly wheel (additional inertia).

A further investigation would be worth understanding on the application of this model in determining the damping and synchronizing torque coefficients for a wide range of motors with different power ratings for several speed variations range. This would allow general trends in the damping and synchronous torque coefficients to be understood and possibly enable simplistic empirically based values to be recommended for a range of industrial motors.

A detailed study on choosing the optimum inertia to reduce the pulsating speed to within safe limits for a range of general industrial dynamic loads could also be done using this model along with a formal optimization method.

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