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UNDERSTANDING THE MATHEMATICAL MODEL FOR THE DISPERSION OF AIR POLLUTANTS

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Abstract:

Air pollution is actually a growing concern of the most nations in the world whether it's created or perhaps developing. The growing number of cars, growing number of industrial plants and less use of recycling of industrial waste materials are actually the fundamental reasons of air pollution. Emission of various forms of dangerous gases and particles are actually the primary root cause of environmental pollution. Because of the environmental pollution, human beings, animals and plants are actually facing various kinds of problem.

Keywords: Air, pollution, environment, human, being, etc.

1. INTRODUCTION

In order to defend ourselves from contaminants released to the environment, it's far better to recognize the physical event engaged in the atmospheric pollutant dispersion. To be able to decrease the pollutants emission in the environment it's always to keep an eye on air quality constantly. Therefore, accurate modelling of pollutants concentration near earth surroundings is drastically important. Atmospheric dispersion of pollutants has attracted researchers in ways that are many. Some people have concentrated in the environmental impact and health hazards, while others have worked on a variety of modeling elements like meteorological conditions, dispersion mechanism, removal mechanisms, topographical characteristics, etc. Invariably, mathematical modeling has been the case of the most of these studies. Many models have been talked about in the past to cope with air pollution dispersion under different atmospheric conditions. Sharan et al. (2003) have provided an overview of mathematical modeling framework of atmospheric dispersion. Agarwal et al. (2008) have given an analytical design to the difficulty of dispersion of an air pollutant with variable wind velocity. Srivastava et al. (2009) have provided a 3 dimensional atmospheric diffusion model with adjustable removal rate and variable wind velocity by using power law profile. Sharan and Kumar (2010) have given an analytical item for the dispersion of pollutant discharged from a constant source in the atmospheric boundary layer describing the crosswind integrated concentration. They've reported that the product may also be used for the focus division of a pollutant produced from a line source perpendicular to the path of the mean wind as well as to learn the turbulent dispersion from a constant 2 dimensional horizontal tool in a wide open channel for a generalized profile of eddy diffusivity and wind flow. Verma et al. (2011) have provided an analytical approach to the difficulty of dispersion of an air pollutant with variable wind velocity as well as variable eddy diffusivity. They've taken wind velocity in the form of wave function. Sharan et al. (1996) have suggested a steady state mathematical model for the dispersion of air pollutants in winds that are low by taking into account the diffusion in the 3 coordinate directions and advection along the mean wind. Verma (2011) has suggested an analytical approach to the difficulty of dispersion of an air pollutant with constant wind velocity and constant removal fee, exactly where it's discovered that the fifty three concentration profile of air pollutant becomes high near the soil but as the distance from the soil increases, the focus profile decreases regularly. Khaled et al. (2011) have given a mathematical type of the atmospheric diffusion equation and have analyzed the advection diffusion equation in 2 directions to get the crosswind integrated concentration. The outcome of different parameters on the associated where , y, z are the cartesian co-ordinates, ( ) is the variable wind velocity which varies with the downwind distance and it is assumed in the form

$$U(x)\frac{\partial C}{\partial x} = K_y\frac{\partial^2 C}{\partial y^2} + K_z\frac{\partial^2 C}{\partial z^2} - \alpha C \quad (1)$$

of a wave function given by ( ) = [1 + (2 /)], where is mean wind velocity is the wave length, C is actually the focus of the, air pollutant, are actually the eddy diffusivities in z- and y- directions respectively which are actually assumed to be constants and is actually the removal price of the air pollutant as a result of several organic mechanism as chemical reaction, which is also taken to be regular. Typically

> in the atmospherethe boundary conditions for the equation (1) are taken as follows

$$K_z\frac{\partial C}{\partial z} = v_d\, C, \qquad z = H \quad (5)$$

$$C(x,y,z) = \frac{Q\delta(y)\delta(z-h_s)}{U(x)}, \; x = 0, 0 \le h_s \le H \quad (2)$$

$$C(x,y,z) = 0, \qquad y \to \pm\infty \quad (3)$$

$$C(x,y,z) = 0, \qquad z = 0 \quad (4)$$

results is actually provided by many researchers, though no attempts have been created with adjustable wind velocity in wave function form and constant removal rate in the constant state condition. Consequently, in this chapter, we've created an effort to the formula of dispersion situation for the focus of air pollutants emitted from a point source with variable wind velocity in wave function form and constant removal rate in the constant state quality, where eddy diffusivity coefficients are actually taken as constant.

2. MATHEMATICAL FORMULATION

The dispersion of an air pollutant in the ambiance under steady state condition is actually discussed by the following partial differential equation

where is actually the Dirac delta function, is actually the emission power of an elevated point source, hs is actually the stack height and vd is actually the deposition velocity of the air pollutant. Condition (2) states that the air pollutant is actually released from the elevated point source of strength Q. Condition (3) states that the focus of the air pollutant is actually zero for → ±∞. Condition (4) states that the concentration of the air pollutant is zero at the ground surface and condition (5) states that there is some diffusion flux at the vertical height H from the ground surface.

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3. METHOD OF SOLUTION

Before remedy, the partial differential equation (1) describing the dispersion of the boundary and air pollutant circumstances are actually made non-dimensional by introducing the following non dimensional quantities

$$x^* = \frac{K_{z_o} x}{U_0 H^2}\ ,\ y^* = \frac{y}{H}\ ,\ z^* = \frac{z}{H}\ ,\ U^* = \frac{U(x)}{U_0}\ , C^* = \frac{U_0 H^2 C}{Q}$$

$$\alpha^* = \frac{H^2 \alpha}{K_{z_o}}\ ,\ \beta^* = \frac{K_y}{K_{z_o}},\ \gamma^* = \frac{K_z}{K_{z_o}},\ \delta(y^*) = H\delta(y),\ N^* = \frac{H v_d}{K_z}$$

Where U0 is the reference wind velocity and On dropping asterisk (*), the equation (1) and is the reference diffusivity. the boundary conditions (2) - (5) may be put in non dimensional form as given below

$$U_0[1 + \epsilon \cos\left(\frac{2\pi x}{\lambda}\right)] \frac{\partial C}{\partial x} = \beta \frac{\partial^2 C}{\partial y^2} + \gamma \frac{\partial^2 C}{\partial z^2} - \alpha c \qquad \textbf{(6)}$$

$$C = \frac{\delta(y)\delta(z-h_s)}{U_0\left[1+\epsilon \cos\left(\frac{2\pi x}{\lambda}\right)\right]}\ ,\quad x = 0 \qquad \textbf{(7)}$$

$$C = 0\ ,\quad y \to \pm\infty \qquad \textbf{(8)}$$

$$C = 0\ ,\quad z = 0 \qquad \textbf{(9)}$$

$$\frac{\partial C}{\partial z} = NC\ ,\quad z = 1 \qquad \textbf{(10)}$$

We solve equation (6) with boundary conditions (7) − (10) by applying Fourier transform technique.

Thus, by taking Fourier transform of (6) with respect to y, we get

$$U_0[1 + \epsilon \cos(\frac{2\pi x}{\lambda})] \frac{\partial \bar{C}}{\partial x} = -(\alpha + p^2\beta)\bar{C} + \gamma \frac{\partial^2 \bar{C}}{\partial z^2} \qquad \textbf{(11)}$$

Where ‽ ≡ ‽(, $,)is the Fourier transform of ! ≡ !(, $,) where ! to y and p is the Fourier transform parameter?

Again, taking Fourier transforms of (6) − (10), the boundary conditions become corresponding

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$$\bar{C} = \frac{\delta(z-h_s)}{U_0[1+\epsilon\cos\left(\frac{2\pi x}{\lambda}\right)]} \quad , \qquad x = 0 \tag{12}$$

$$\bar{C} = 0 \quad , \qquad y \to \pm\infty \tag{13}$$

$$\bar{C} = 0 \; , \qquad z = 0 \tag{14}$$

$$\frac{\partial \bar{C}}{\partial z} = N\bar{C} \; , \qquad z = 1 \tag{15}$$

Again, to solve equation (11), we shall use the method of separation of variables and thus we assume the trial solution of equation (11) in the following form

$$\bar{C} = X(x)Z(z) \tag{16}$$

where%( ) is a function of only and &( ) is a function of z only. Using (16) in equation (11), we get

$$U_0[(1+\epsilon\cos\left(\frac{2\pi x}{\lambda}\right)]\frac{\partial}{\partial x}\{X(x)Z(z)\}$$

$$= -(\alpha+\beta p^2)X(x)Z(z) + \gamma\frac{\partial^2}{\partial z^2}\{X(x)Z(z)\} \tag{17}$$

Which can also be written in the following form

$$U_0\{(1+\epsilon\cos\left(\frac{2\pi x}{\lambda}\right)\}X'Z = -(\alpha+p^2\beta)XZ + \gamma Z''X \tag{18}$$

Dividing both sides by%&, we have

$$\frac{U_0\{(1+\epsilon\cos\left(\frac{2\pi x}{\lambda}\right)\}X'}{X} = -(\alpha+p^2\beta) + \frac{\gamma Z''}{Z} \tag{19}$$

which may be put in the following form:

$$\frac{Z''}{Z} = [\frac{U_0\{(1+\epsilon\cos\left(\frac{2\pi x}{\lambda}\right)\}}{\gamma}]\frac{X'}{X} + \frac{(p^2\beta+\alpha)}{\gamma} = -\lambda^2 \text{ , say} \tag{20}$$

where λ^2 is a separation constant.

Taking the first and third ratios of equation (20), we have

$$Z''+\lambda^2 Z=0 \quad \textbf{(21)}$$

whose solution is given by

$$Z(z)=A\cos(\lambda z)+B\sin(\lambda z) \quad \textbf{(22)}$$

4. RESULTS AND DISCUSSION

In order to find the outcome of different parameters on dispersion of air pollutant, the focus profile in non dimensional form is actually estimated by equation (34). The parametric values used in the evaluation are actually taken as follow

$$\alpha = 2,\ \beta = 10,\quad \gamma = 1,\quad h_s = 0.2,\ H = 1,\ \epsilon = 0.005,\ \lambda = 1, U_0 = 1 \quad \textbf{(34)}$$

In figure 1, the concentration profile is plotted against the downwind distance (0 ≤ ≤ 1) for different values of vertical distances (= 0.2, 0.4, 0.6) and the value of y is taken to be zero. It is seen that the concentration profile decreases continuously with increasing downwind distance and it becomes negligible at =1.

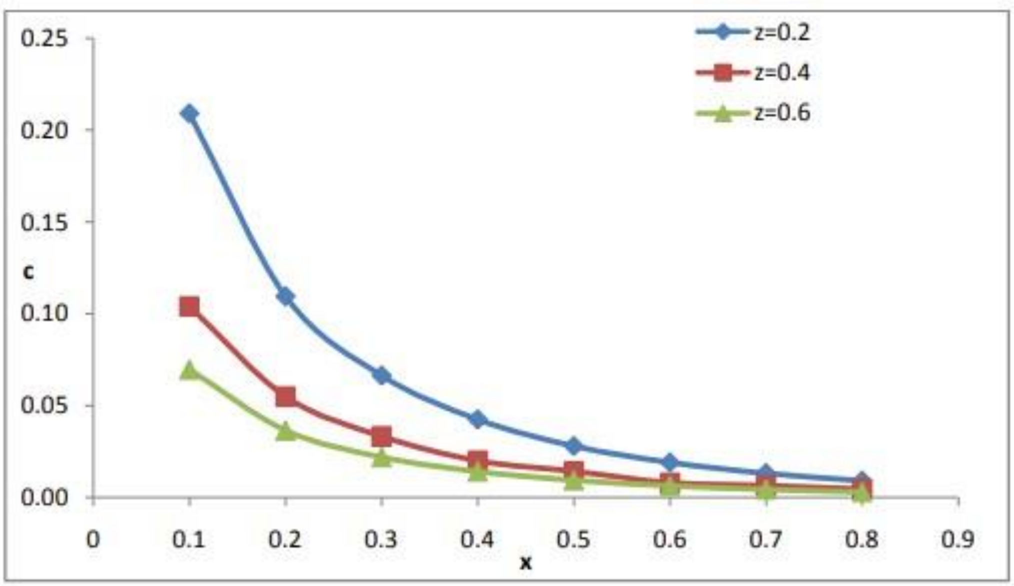

Fig.1 Variation of concentration profile with downwind distance ' for different values of vertical distance z when crosswind distance y=0.

It is also observed that the concentration profile decreases with increasing vertical distances (= 0.2, = 0.4, = 0.6) and the decrease in concentration with vertical distance being higher at lower values of downwind distance and at higher values of downwind distance, there is

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negligible change in the concentration of air pollution with increasing vertical distance.

In figure 2, the concentration profile is plotted against the downwind distance *(0 ≤ ≤ 1)* for different values of vertical distances *(= 0.2, 0.4, 0.6)* and the value of y is taken to be one. Here also, it is seen that the concentration profile decreases continuously with increasing downwind distance and it becomes negligible at = 1.

It is also observed that the concentration profile decreases with increasing vertical distances *(= 0.2, = 0.4, = 0.6)* and the reduction in focus with vertical distance being higher at lower values of downwind distance and at greater values of downwind distance, there's negligible change in the focus of air pollution with increasing vertical distance.

5. CONCLUSION

With this product, it's discovered that the focus

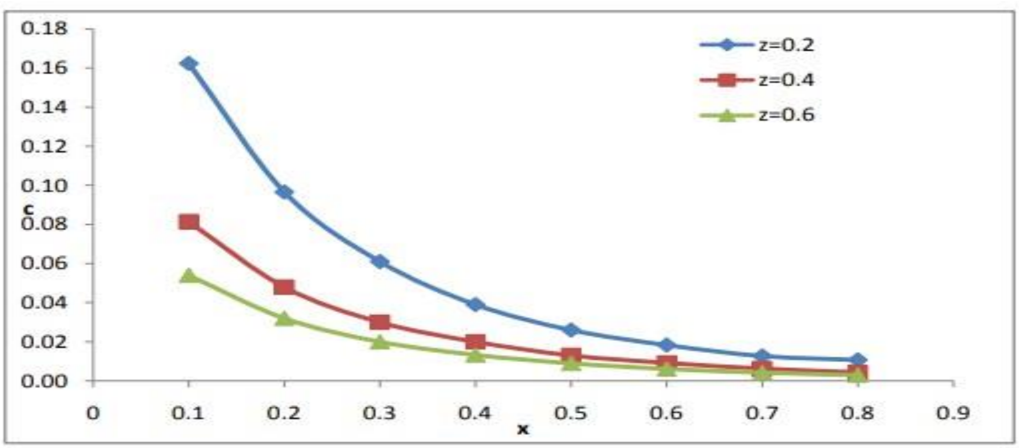

Fig.2: Variation of concentration profile with downwind distance ' for different values of vertical distance z when crosswind distance y=1.

profile of air pollutant decreases continuously with increasing downwind distance and it gets to be negligible at =1. It's also seen that the focus profile decreases with increasing the decrease and vertical distances in focus with vertical distance being higher at lower values of downwind distance and at greater values of downwind distance, there's negligible change in the focus of air pollution with increasing vertical distance

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