FUZZY SUPRA STRONGLY BAIRE SPACES

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ABSTRACT

In this paper, introduced and studied the concept of fuzzy supra strongly nowhere dense sets and fuzzy supra strongly Baire spaces. Several characterizations of fuzzy supra strongly nowhere dense sets and fuzzy supra strongly Baire spaces are obtained. Also illustrate these concepts with suitable example.

Keywords: Fuzzy Supra dense set, Fuzzy Supra nowhere dense set, Fuzzy Supra Baire space, Fuzzy Supra first category space, Fuzzy Supra residual set, Fuzzy Supra submaximal spaces.

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1. INTRODUCTION

In 1965, L.A. Zadeh [10] introduced the concept of fuzzy sets as a new approach for modeling uncertainties. The potential of fuzzy notion was realized by the researchers and has successfully been applied in all branches of Mathematics. In 1968 C.L. Chang [3] was introduced the concept of fuzzy topological space. The concept of Baire spaces in fuzzy setting was introduced and studied by G.Thangaraj and S.Anjalmose in [9]. The concept of fuzzy supra Baire spaces introduced and studied by this author in [7]. The purpose of this paper is to introduce and study a new class of fuzzy supra topological spaces(in short FSTS) called fuzzy supra strongly Baire spaces. In this paper, the notions of fuzzy supra strongly nowhere dense sets and fuzzy supra strongly Baire spaces are introduced. Several characterizations of fuzzy supra strongly nowhere dense sets and fuzzy supra strongly Baire spaces are established.

2. PRELIMINARIES

Definition 2.1[2].

A collection δ^* of fuzzy sets in a set X is called fuzzy supra topology on X if the following conditions are satisfied:

1) **0** and **1** belongs to δ^*.

2) $g_\chi \in \delta^*$ for each $\chi \in \Lambda$ implies $(\vee_{\chi \in \Lambda} g_\chi) \in \delta^*$.

The pair (X, δ^*) is called a FSTS. The elements of δ^* are called fuzzy open sets and the complement of a fuzzy open set is called fuzzy supra closed set.

Lemma 2.1 [5]

For a fuzzy set χ in a fuzzy topological space X,

(i) $1 - int^*(\chi) = cl^*(1 - \chi)$,

(ii) $1 - cl^*(\chi) = int^*(1 - \chi)$.

Definition 2.2 [6]

A fuzzy open set χ in FSTS (X,T^*) is called fuzzy supra F_σ-set in (X,T^*) if $\chi = \vee_{i=1}^{\infty}(\chi_i)$, where $1 - \chi_i \in T^*$ for $i \in I$,

Definition 2.3 [6]

A fuzzy open set χ in FSTS (X,T^*) is called fuzzy supra G_σ - set in (X,T^*) if $\chi = \wedge_{i=1}^{\infty}(\chi_i)$, where $\chi_i \in T^*$ for $i \in I$.

Definition 2.4 [6]

A fuzzy set χ in a FSTS (X,T^*) is called a FS dense if there exists no FS closed set β in (X,T^*) such that $\chi < \beta < 1$. That is, $cl^*(\chi)=1$, in (X,T^*).

Definition 2.5 [6]

A fuzzy set χ in FSTS (X,T^*) is called a fuzzy supra nowhere dense set if there exists no non-zero fuzzy open set μ in (X,T^*) such that $\mu < cl^*(\chi)$. That is, $int^*cl^*(\chi)=0$, in (X,T^*).

Definition 2.6 [7]

In a FSTS (X,T^*) a fuzzy set χ is said to be a fuzzy supra first category set if $\chi = \wedge_{i=1}^{\infty}(\chi_i)$,where$(\chi_i)$'s are fuzzy supra nowhere dense sets in (X,T^*). Any other fuzzy set in (X,T^*) is said to be a fuzzy supra second category set in (X,T^*).

Definition 2.7 [7]

In a FSTS (X,T^*) is called a fuzzy supra submaximal space if for each fuzzy set in (X,T^*) such that $cl^*(\chi) = 1$, then $\chi \in T^*$ in (X, T^*).

Definition 2.8 [8]

A fuzzy supra set χ in a FSTS (X,T^*) is called a fuzzy supra strongly first category set $\chi = \vee_{i=1}^{\infty}(\chi_i)$, where (χ_i)'s are fuzzy supra strongly nowhere dense sets in (X,T^*). Any other fuzzy supra set in (X,T^*) is said to be a fuzzy supra strongly second category set in (X,T^*).

Definition 2.9 [8]

If χ is a fuzzy supra strongly first category set in a FSTS (X,T^*), then $1-\chi$ is a fuzzy supra strongly residual set in (X,T^*)

Definition 2.10 [8]

A FSTS (X,T^*) is called a fuzzy supra strongly first category space, if the fuzzy set 1_X is a fuzzy supra strongly first category set in (X,T^*). That is, $1_X = \vee_{i=1}^{\infty}(\chi_i)$, where (χ_i)'s are fuzzy supra strongly nowhere dense sets in (X,T^*). Otherwise (X,T^*) will be called a fuzzy supra strongly second category space.

Theorem 2.1 [7]

Let (X,T^*) be a FSTS. Then the following are equivalent:

(i) (X,T^*) is a fuzzy supra Baire space.

(ii) $int^*(\chi)=0$, for every fuzzy supra first category set in χ in (X,T^*).

(iii) $cl^*(\mu)=1$, for every fuzzy supra residual set μ in (X,T^*).

3. FUZZY SUPRA STRONGLY NOWHERE DENSE SET

Definition 3.1

In a FSTS (X,T^*) is called a fuzzy supra strongly nowhere dense set, if $\chi\wedge(1-\chi)$ is a fuzzy supra nowhere dense set in (X,T^*). That is $int^*\{cl^*[\chi\wedge(1-\chi)]\}=0$, in (X,T^*).

Proposition 3.1.

If χ is a fuzzy supra nowhere dense set in a FSTS (X,T^*), then χ is a fuzzy supra strongly nowhere dense set in (X,T^*).

Proof.

Let χ be a fuzzy supra nowhere dense set in (X,T^*). Then $int^*cl^*(\chi)=0$ in (X,T^*). Since $\chi\wedge(1-\chi)\leq\chi$ in (X,T^*), $int^*cl^*[\chi\wedge(1-\chi)]\leq int^*cl^*(\chi)$ and hence $int^*cl^*[\chi\wedge(1-\chi)]\leq 0$: That is, $int^*cl^*[\chi\wedge(1-\chi)]=0$. Hence, χ is a fuzzy supra strongly nowhere dense set in (X,T^*).

Example 3.1.

Let $X=\{a,b,c\}$.Then the fuzzy sets α, β, γ are defined on (X,T^*) as follows:

$\alpha: X\to[0,1]$ defined as $\alpha(a)=0.6$; $\alpha(b)=0.5$; $\alpha(c)=0.7$

$\beta: X\to[0,1]$ defined as $\beta(a)=0.7$; $\beta(b)=0.6$; $\beta(c)=0.8$

$\gamma: X\to[0,1]$ defined as $\gamma(a)=0.7$; $\gamma(b)=0.7$; $\gamma(c)=0.8$

Then $T^*=\{0, \alpha, \beta, \gamma, \alpha\vee\beta, \alpha\vee\gamma, \beta\vee\gamma, \alpha\wedge\beta, \beta\wedge\gamma, \alpha\wedge\gamma, 1\}$ is a fuzzy supra topology on X. The non-zero fuzzy supra open sets in (X,T^*) are $(\alpha, \beta, \gamma, \alpha\vee\beta, \alpha\vee\gamma, \beta\vee\gamma, \alpha\wedge\beta, \beta\wedge\gamma, \alpha\wedge\gamma)$. Now, $int^*cl^*[\gamma\wedge(1-\gamma)]= int^*cl^*[\beta\wedge(1-\beta)]= int^*cl^*[(\alpha\vee\beta)\wedge(1-\alpha\vee\beta)]=0$. Hence, γ, β, $\alpha\vee\beta$ are fuzzy supra strongly nowhere dense sets in (X,T^*).

Remark 3.1.

A fuzzy supra strongly nowhere dense set in a FSTS (X,T^*) need not be a fuzzy supra nowhere dense set in (X,T^*). For, in example 3.1, α is a fuzzy supra strongly nowhere dense set, but not a fuzzy supra nowhere dense set in (X,T^*).

Proposition 3.2.

If $int^*(\chi)$ is a fuzzy supra dense set in a FSTS (X,T^*), then χ is a fuzzy supra strongly nowhere dense set in (X,T^*).

Proof.

Let $int^*(\chi)$ is a fuzzy supra dense set in (X,T^*). Then $cl^*[int^*(\chi)]=1$, in (X,T^*) and $1-cl^*[int^*(\chi)]=0$. This implies that $int^*cl^*(1-\chi)=0$ in (X,T^*). Since $\chi\wedge(1-\chi)\leq 1-\chi$ in (X,T^*), $int^*cl^*[\chi\wedge(1-\chi)]\leq int^*cl^*(1-\chi)=0$ and hence $int^*cl^*[\chi\wedge(1-\chi)]=0$. Hence χ is a fuzzy supra strongly nowhere dense set in (X,T^*).

Proposition 3.3.

If $1-\chi$ is a fuzzy supra nowhere dense set in a FSTS (X,T^*), then χ is a fuzzy supra strongly nowhere dense set in (X,T^*)

Proof.

Suppose that $1-\chi$ is a fuzzy supra nowhere dense set in (X,T^*) Then, $int^*cl^*(1-\chi) = 0$ in (X,T^*). Since $\chi\wedge(1-\chi) \leq 1-\chi$, $int^*cl^*[\chi\wedge(1-\chi)] \leq int^*cl^*(1-\chi)$ and hence $int^*cl^*[\chi\wedge(1-\chi)] \leq 0$. That is., $int^*cl^*[\chi\wedge(1-\chi)] = 0$ in (X,T^*) and hence χ is a fuzzy supra strongly nowhere dense set in (X,T^*).

Proposition 3.4.

If $cl^*[int^*(1-\chi)] = 1$ in a FSTS (X,T^*), then χ is a fuzzy supra strongly nowhere dense set in (X; T).

Proof.

Suppose that $cl^*[int^*(1-\chi)] = 1$ in (X,T^*), Then $1-cl^*[int^*(1-\chi)] = 0$ and $1-\{1-int^*[cl^*(\chi)]\} = 0$. This implies that $int^*[cl^*(\chi)] = 0$ in (X,T^*). Thus χ is a fuzzy supra nowhere dense set in (X,T^*) Then, by proposition 3.1, χ is a fuzzy supra strongly nowhere dense set in (X,T^*)

Proposition 3.5.

If χ is a fuzzy supra strongly nowhere dense set in a FSTS (X,T^*), then $(1-\chi)$ is also a fuzzy supra strongly nowhere dense set in (X,T^*).

Proof.

Let χ be a fuzzy supra strongly nowhere dense set in (X,T^*) Then $int^*cl^*[\chi\wedge(1-\chi)] = 0$ in (X,T^*). Now $int^*cl^*\{(1-\chi)\wedge[1-(1-\chi)]\} = int^*cl^*[(1-\chi)\wedge\chi]=0$ and hence $int^*cl^*\{(1-\chi)\wedge[1-(1-\chi)]\} = 0$. This implies that $(1-\chi)$ is a fuzzy supra strongly nowhere dense set in (X,T^*).

Proposition 3.6.

If χ is a fuzzy supra nowhere dense set in a FSTS (X,T^*), then $1-\chi$ is a fuzzy supra strongly nowhere dense set in (X,T^*).

Proof.

Let χ be a fuzzy supra nowhere dense set in (X,T^*) Then, by proposition 3.1, χ is a fuzzy supra strongly nowhere dense set in (X,T^*) and by proposition 3.5, $1-\chi$ is a fuzzy supra strongly nowhere dense set in (X,T^*).

Proposition 3.7.

If χ is a fuzzy supra strongly nowhere dense set in a FSTS (X,T^*), then $cl^*(\chi)\vee cl^*(1-\chi) = 1$, in (X,T^*).

Proof.

Let χ be a fuzzy supra strongly nowhere dense set in (X,T^*). Then $int^*cl^*[\chi\wedge(1-\chi)] = 0$ in (X,T^*). Now, $1-int^*cl^*[\chi\wedge(1-\chi)] = 1$ and hence $cl^*int^*[1-\{\chi\wedge(1-\chi)\}] = 1$, in (X,T^*). But, $cl^*int^*[1-\{\chi\wedge(1-\chi)\}] \leq cl^*[1-\{\chi\wedge(1-\chi)\}]$ implies that $1 \leq cl^*[1-\{\chi\wedge(1-\chi)\}]$. Thus, $cl^*[1-\{\chi\wedge(1-\chi)\}] = 1$, in (X,T^*). This implies that $cl^*[(1-\chi)\vee\chi] = 1$ in (X,T^*). But, $cl^*[(1-\chi)\vee\chi] = cl^*(1-\chi)\vee cl^*(\chi)$. Hence, $cl^*(\chi)\vee cl^*(1-\chi) = 1$ in (X,T^*).

Definition 3.2

Let χ be a fuzzy set in a FSTS (X,T^*). The fuzzy supra boundary of χ is defined as $Bd(\chi) = cl^*(\chi)\wedge cl^*(1-\chi)$.

Proposition 3.8.

If χ is a fuzzy set defined on X such that $int^*[Bd(\chi)] = 0$ in a FSTS (X,T^*), then χ is a fuzzy supra strongly nowhere dense set in (X,T^*)

Proof.

Let χ be a fuzzy set, such that $int^*[Bd(\chi)] = 0$ in (X,T^*). Since $Bd(\chi) = cl^*(\chi) \wedge cl^*(1-\chi)$ and $cl^*(\chi) \wedge cl^*(1-\chi) \geq cl^*[\chi \wedge (1-\chi)]$, we have $Bd(\chi) \geq cl^*[\chi \wedge (1-\chi)]$ and hence $int^*cl^*[\chi \wedge (1-\chi)] \leq int^*[Bd(\chi)]$ in (X,T^*) Then $int^*cl^*[\chi \wedge (1-\chi)] \leq 0$. That is., $int^*cl^*[\chi \wedge (1-\chi)] = 0$ and hence χ is a fuzzy supra strongly nowhere dense set in (X,T^*).

Definition 3.3

A fuzzy set χ in a FSTS (X,T^*) is called a fuzzy supra simply open set if $Bd(\chi)$ is a fuzzy supra nowhere dense set in (X,T^*). That is, χ is a fuzzy supra simply open set in (X,T^*) if $int^*cl^*[Bd(\chi)] = 0$ in (X,T^*).

Example 3.2

Let X={a,b,c}. The fuzzy sets α,β and γ are defined on X as follows;

$\alpha: X \to [0,1]$ defined as $\alpha(a)=0.8$; $\alpha(b)=0.6$; $\alpha(c)=0.7$

$\beta: X \to [0,1]$ defined as $\beta(a)=0.7$; $\beta(b)=0.8$; $\beta(c)=0.4$

$\gamma: X \to [0,1]$ defined as $\gamma(a)=0.5$; $\gamma(b)=0.7$; $\gamma(c)=0.8$

Then $T^*=\{0, \alpha, \beta, \gamma, \alpha\vee\beta, \alpha\vee\gamma, \beta\vee\gamma, \alpha\wedge\beta, \beta\wedge\gamma, \alpha\wedge\gamma, 1\}$ is a fuzzy supra topology on (X,T^*). $int^*cl^*[Bd(\beta)] = int^*cl^*[cl^*(\beta) \wedge cl^*(1-\beta)] = int^*cl^*[1 \wedge (1-\beta)] = int^*cl^*(1-\beta) = int^*(1-\beta) = 1-cl^*(\beta) = 1-1 = 0$ in (X,T^*).Hence β is a fuzzy simply open set in (X,T^*).

Proposition 3.9.

If χ is a fuzzy supra simply open set in a FSTS (X,T^*) then χ is a fuzzy supra strongly nowhere dense set in (X,T^*).

Proof.

Let χ be a fuzzy supra simply open set in (X,T^*). Then $int^*cl^*[Bd(\chi)] = 0$ in (X,T^*). But $int^*[Bd(\chi)] \leq int^*cl^*[Bd(\chi)]$ implies that $int^*[Bd(\chi)] = 0$ in (X,T^*) Then, by proposition 3.8, χ is a fuzzy supra strongly nowhere dense set in (X,T^*).

Proposition 3.10.

If χ is a fuzzy closed set with $int^*(\chi) = 0$ in a FSTS (X,T^*), then χ is a fuzzy supra strongly nowhere dense set in (X,T^*).

Proof.

Let χ be a fuzzy closed set with $int^*(\chi) = 0$ in (X,T^*)Then, $int^*cl^*[cl^*(\chi) \wedge cl^*(1-\chi)] = int^*cl^*[\chi \wedge (1-int^*(\chi))] = int^*cl^*[\chi \wedge 1] = int^*cl^*(\chi) = int^*(\chi) = 0$, and hence χ is a fuzzy supra simply open set in (X,T^*). By proposition 3.9, χ is a fuzzy supra strongly nowhere dense set in (X,T^*).

Proposition 3.11.

If χ is a fuzzy open and fuzzy supra dense set in a FSTS (X,T^*), then χ is a fuzzy supra strongly nowhere dense set in (X,T^*).

Proof.

Let χ be a fuzzy open and fuzzy supra dense set in (X,T^*). Then $1-\chi$ is a fuzzy supra closed set with $int^*(1-\chi) = 1-cl^*(\chi) = 1-1 = 0$ in (X,T^*). Then, by proposition 3.10, $1-\chi$ is a fuzzy supra strongly nowhere dense set in (X,T^*) and by proposition 3.5, $1-(1-\chi)$ is a fuzzy supra strongly nowhere dense set in (X,T^*) and thus χ is a fuzzy supra strongly nowhere dense set in (X,T^*)

Proposition 3.12.

If χ is a fuzzy supra first category set in a FSTS (X,T^*), then χ is a fuzzy supra strongly first category set in (X,T^*).

Proof.

Let χ be a fuzzy supra first category set in (X,T^*). Then $\chi = \vee_{i=1}^{\infty}(\chi_i)$, where (χ_i)'s are fuzzy supra nowhere dense sets in (X,T^*). By proposition 3.1, the fuzzy supra nowhere dense sets (χ_i)′s are fuzzy supra strongly nowhere dense sets in (X,T^*) and hence $\chi = \vee_{i=1}^{\infty}(\chi_i)$, where (χ_i)′s are fuzzy supra strongly nowhere dense sets in (X,T^*), implies that χ is a fuzzy supra strongly first category set in (X,T^*).

Proposition 3.13.

If $\chi = \vee_{i=1}^{\infty}(\chi_i)$ where (χ_i)′s are fuzzy supra closed sets with $int^*(\chi_i) = 0$ in (X,T^*), then χ is a fuzzy supra strongly first category set in (X,T^*).

Proof.

Suppose that $\chi = \vee_{i=1}^{\infty}(\chi_i)$ where $1-\chi_i \in T$ and $int^*(\chi_i) = 0$ in (X,T^*). Now by proposition 3.10, the fuzzy supra closed sets (χ_i)′s with $int^*(\chi_i) = 0$, are fuzzy supra strongly nowhere dense sets in (X,T^*) and hence $\chi = \vee_{i=1}^{\infty}(\chi_i)$, where (χ_i)′s are fuzzy supra strongly nowhere dense sets in (X,T^*), implies that χ is a fuzzy supra strongly first category set in (X,T^*).

Proposition 3.14.

If $\chi = \vee_{i=1}^{\infty}(\chi_i)$, where (χ_i)′s are fuzzy supra simply open sets in a FSTS (X,T^*), then χ is a fuzzy supra strongly first category set in (X,T^*).

Proof.

Suppose that $\chi = \vee_{i=1}^{\infty}(\chi_i)$, where (χ_i)′s are fuzzy supra simply open sets in (X,T^*). By proposition 3.9, the fuzzy supra simply open sets (χ_i)′s are fuzzy supra strongly nowhere dense sets in (X,T^*) and hence χ is a fuzzy supra strongly first category set in (X,T^*).

Proposition 3.15.

If $\chi = \vee_{i=1}^{\infty}(\chi_i)$, where $int^*[Bd(\chi_i)] = 0$ in (X,T^*), then χ is a fuzzy supra strongly first category set in (X,T^*).

Proof.

The proof follows from proposition 3.8.

4. FUZZY SUPRA STRONGLY BAIRE SPACE

Definition 4.1

In a FSTS (X,T^*) is called a fuzzy supra strongly Baire space if $cl^*(\wedge_{i=1}^{\infty}(\chi_i))=1$, where (χ_i)'s are fuzzy strongly nowhere dense sets in (X,T^*)**.**

EXAMPLE 4.1.

Let X={a,b,c}.Then the fuzzy sets α, β, γ are defined on (X,T^*)as follows:

$\alpha: X\to[0,1]$ defined as $\alpha(a)=0.8$; $\alpha(b)=0.7$; $\alpha(c)=0.3$

$\beta: X\to[0,1]$ defined as $\beta(a)=0.6$; $\beta(b)=0.5$; $\beta(c)=0.9$

γ: X→[0,1] defined as $\gamma(a)=0.6$; $\gamma(b)=0.4$; $\gamma(c)=0.7$

Then $T^*=\{0, \alpha, \beta, \gamma, \alpha \vee \beta, \beta \vee\gamma, \alpha \vee\gamma, \alpha \wedge \beta, \beta \wedge \gamma, \alpha \wedge\gamma, 1\}$ is a fuzzy supra topology on (X,T^*).On computation $int^*cl^*[\alpha\wedge(1-\alpha)]=0$, $int^*cl^*[(\alpha \wedge \beta)\wedge(1-(\alpha \wedge \beta))]=0$, $int^*cl^*[(\beta \vee\gamma)\wedge(1-(\beta \vee\gamma)]=0$, $int^*cl^*[(\alpha \vee\gamma)\wedge(1-(\alpha \vee\gamma)]=0$.Therefore the fuzzy supra strogly nowhere dense sets are α, $\alpha \wedge \beta$, $\beta \vee\gamma$, $\alpha \vee\gamma$, $1-\alpha$, $1-(\alpha \wedge \beta)$, $1-(\beta \vee\gamma)$, $1-(\alpha \vee\gamma)$.Then $cl^*\{\alpha\vee (\alpha \wedge \beta) \vee(\beta \vee\gamma) \vee (\alpha \vee\gamma) \vee (1-\alpha) \vee [1-(\alpha \wedge \beta)] \vee [1-(\beta \vee\gamma)] \vee [1-(\alpha \vee\gamma)]=1$.Hence (X,T^*) is called a fuzzy supra strongly Baire space.

Proposition 4.1.

Let (X,T^*) be a FSTS. Then, the following are equivalent:

(i). (X,T^*) is a fuzzy supra strongly Baire space.

(ii). $cl^*(\chi) = 1$, for each fuzzy supra strongly first category set χ in (X,T^*)

(iii). $int^*(\mu) = 0$, for each fuzzy supra strongly residual set μ in (X,T^*).

Proof.

(i) → (ii)

Let χ be a fuzzy supra strongly first category set in (X,T^*). Then $\chi = \vee_{i=1}^{\infty}(\chi_i)$, where (χ_i)′s are fuzzy supra strongly nowhere dense sets in (X,T^*). Since (X,T^*) is a fuzzy supra strongly Baire space, $cl^*[\vee_{i=1}^{\infty}(\chi_i)] = 1$ and hence $cl^*[\chi] = 1$, in (X,T^*).

(ii) → (iii)

Let μ be a fuzzy supra strongly residual set in (X,T^*). Then $1-\mu$ is a fuzzy supra strongly first category set in (X,T^*). By hypothesis, $cl^*(1-\mu) = 1$ in (X,T^*) .Then $1- int^*(\mu) = cl^*(1-\mu) = 1$ and hence $int^*(\mu) = 0$ in (X,T^*).

(iii) → (i)

Let χ be a fuzzy supra strongly first category set in (X,T^*). Then $\chi = \vee_{i=1}^{\infty}(\chi_i)$, where (χ_i)′s are fuzzy supra strongly nowhere dense sets in (X,T^*). Since χ is a fuzzy supra strongly first category set in (X,T^*) $1-\chi$ is a fuzzy supra strongly residual set in (X,T^*). By hypothesis, $int^*(1-\chi) = 0$ in (X,T^*). Now $1- cl^*(\chi) = int^*(1-\chi) = 0$ implies that $cl^*(\chi) = 1$ and then $cl^*[\vee_{i=1}^{\infty}(\chi_i)] = 1$, where (χ_i)′s are fuzzy supra strongly nowhere dense sets in (X,T^*) implies that (X,T^*) is a fuzzy supra strongly Baire space.

Proposition 4.2.

If (X,T^*) is a fuzzy supra strongly Baire space, then

(i) $cl^*[\vee_{i=1}^{\infty}(\chi_i)] = 1$,where (χ_i)′s are fuzzy supra closed sets and $int^*(\chi_i) = 0$ in (X,T^*).

(ii) $cl^*[\vee_{i=1}^{\infty}(\chi_i)] = 1$, where (χ_i)′s are fuzzy supra simply open sets in (X,T^*).

(iii) $cl^*[\vee_{i=1}^{\infty}(\chi_i)] = 1$, where $int^*[Bd(\chi_i)] = 0$ in (X,T^*).

Proof.

(i) Let $\chi = \vee_{i=1}^{\infty}(\chi_i)$, where (χ_i)′s are fuzzy supra closed sets with $int^*(\chi_i)= 0$. Then, by proposition 3.13, χ is a fuzzy supra strongly first category set in (X,T^*) Since (X,T^*) is a fuzzy supra strongly Baire space, by proposition 4.1, $cl^*(\chi) = 1$ in (X,T^*) Thus $cl^*[\vee_{i=1}^{\infty}(\chi_i)] = 1$, where $(1-\chi_i)\epsilon T$ and $int(\chi_i) = 0$.

(ii) Let $\chi = \vee_{i=1}^{\infty}(\chi_i)$, where (χ_i)′s are fuzzy supra simply open sets in (X,T^*). Then by proposition 3.14, χ is a fuzzy supra strongly first category set in (X,T^*). Since (X,T^*) is a fuzzy supra strongly Baire space, by proposition 4.1, $cl^*(\chi) = 1$ in (X,T^*). Thus $cl^*[\vee_{i=1}^{\infty}(\chi_i)] = 1$, where (χ_i)′s are fuzzy supra simply open sets in a fuzzy supra strongly Baire space (X,T^*).

(iii) Let $\chi = \vee_{i=1}^{\infty}(\chi_i)$, where (χ_i)′s are the fuzzy sets on (X,T^*) with $int^*[Bd(\chi_i)] = 0$ in (X,T^*). Then by proposition 3.15, χ is a fuzzy supra strongly first category set in (X,T^*). Since (X,T^*) is a fuzzy supra strongly Baire space, by proposition 4.1, $cl^*(\chi) = 1$ in (X,T^*). Thus $cl^*[\vee_{i=1}^{\infty}(\chi_i)] = 1$, where $int^*[Bd(\chi_i)] = 0$ in (X,T^*).

Proposition 4.3.

If each fuzzy supra open set is a fuzzy supra dense set in a FSTS (X,T^*), then (X,T^*) is a fuzzy supra strongly Baire space.

Proof.

Let (χ_i)'s be fuzzy supra open sets in (X,T^*). By hypothesis, (χ_i)'s are fuzzy supra dense sets in (X,T^*). Thus (χ_i)'s are fuzzy supra open and fuzzy supra dense sets in (X,T^*). Then by proposition 3.11, (χ_i)'s are fuzzy supra strongly nowhere dense sets in (X,T^*). Let $\chi = \vee_{i=1}^{\infty}(\chi_i)$. Then χ is a fuzzy supra strongly first category set in (X,T^*). Now $cl^*(\chi) = cl^*[\vee_{i=1}^{\infty}(\chi_i)] > \vee_{i=1}^{\infty} cl^*(\chi_i) = \vee_{i=1}^{\infty} (1) = 1$. That is., $cl^*(\chi) = 1$ in (X,T^*). Then, by proposition 4.1, (X,T^*) is a fuzzy supra strongly Baire space.

Proposition 4.4.

If (X,T^*) is a fuzzy supra hyperconnected space, then (X,T^*) is a fuzzy supra strongly Baire space.

Proof.

Let (X,T^*) be a fuzzy supra hyperconnected space. Then each fuzzy supra open set is a fuzzy supra dense set in (X,T^*). Then, by proposition 4.3, (X,T^*) is a fuzzy supra strongly Baire space.

Proposition 4.5.

If (χ_i)′s ($i = 1$ to ∞) are fuzzy supra simply open sets in a fuzzy supra strongly Baire space, then (χ_i)′s are not fuzzy supra dense sets in (X,T^*)

Proof.

Let (χ_i)′s ($i = 1$ to ∞) be fuzzy supra simply open sets in (X,T^*). Suppose that $\chi = \vee_{i=1}^{\infty}(\chi_i)$. Then by proposition 3.14, χ is a fuzzy supra strongly first category set in (X,T^*). Since (X,T^*) is a fuzzy supra strongly Baire space, by proposition 4.1, $cl^*(\chi) = 1$ in (X,T^*). Then $cl^*[\vee_{i=1}^{\infty}(\chi_i)] = 1$ in (X,T^*). But $\vee_{i=1}^{\infty} cl^*(\chi_i) < cl^*[\vee_{i=1}^{\infty}(\chi_i)]$ implies that $\vee_{i=1}^{\infty} cl^*(\chi_i) \neq 1$ in (X,T^*) and hence $cl^*(\chi_i) \neq 1$. Thus the fuzzy supra simply open sets (χ_i)′s are not fuzzy supra dense sets in (X,T^*).

Proposition 4.6.

If χ is a fuzzy supra first category set in a FSTS (X,T^*) then there exists a fuzzy supra strongly first category set μ in (X,T^*) such that $\chi \leq \mu$.

Proof.

Let χ be a fuzzy supra first category set in (X,T^*). Then $\chi = \vee_{i=1}^{\infty}(\chi_i)$, where (χ_i)′s are fuzzy supra nowhere dense sets in (X,T^*). By proposition 3.6, $(1 - \chi_i)$′s are fuzzy supra strongly nowhere dense sets in (X,T^*). Then $\{(\chi_i)\vee(1\ \ \chi_i)\}$ are fuzzy supra strongly nowhere dense sets in (X,T^*). Now $\chi_i \leq (\chi_i)\vee(1-\chi_i)$ implies that $\vee_{i=1}^{\infty}(\chi_i) \leq \vee_{i=1}^{\infty} [\chi_i \vee (1-\chi_i)]$ --- (1). Let $\mu = \vee_{i=1}^{\infty} [\chi_i \vee (1-\chi_i)]$, then μ is a fuzzy supra strongly first category set in (X,T^*). Then from (1), we have $\chi \leq \mu$.

Proposition 4.7.

If δ is a fuzzy supra residual set in a FSTS (X,T^*), then there exists a fuzzy supra strongly residual set η in (X,T^*) such that $\eta \leq \delta$.

Proof.

Let δ be a fuzzy supra residual set in (X,T^*). Then $1-\delta$ is a fuzzy supra first category set in (X,T^*). Then, by proposition 4.6, there exists a fuzzy supra strongly first category set μ in (X,T^*) such that $1-\delta$

$\leq \mu$. Then $1 - \mu \leq \delta$. Let $1-\mu = \eta$ and thus η is a fuzzy supra strongly residual set in (X,T^*). Hence $\eta \leq \delta$ in (X,T^*).

5. FUZZY SUPRA STRONGLY BAIRE SPACES AND FUZZY SUPRA BAIRE SPACES

Proposition 5.1.

If $int^*(\mu) = 0$ for each fuzzy supra strongly first category set μ in a FSTS (X,T^*), then (X,T^*) is a fuzzy supra Baire space.

Proof.

Let χ be a fuzzy supra first category set in (X,T^*). Then by proposition 4.6, there exists a fuzzy supra strongly first category set μ in (X,T^*) such that $\chi \leq \mu$. Then $int^*(\chi) \leq int^*(\mu)$ in (X,T^*). By hypothesis $int^*(\mu) = 0$ in (X,T^*) and thus $int^*(\chi) = 0$ in (X,T^*). Thus, by theorem 2.1, (X,T^*) is a fuzzy supra Baire space.

Proposition 5.2.

If $cl^*(\eta) = 1$, for each fuzzy supra strongly residual set η in a FSTS (X,T^*), then (X,T^*) is a fuzzy supra Baire space.

Proof.

Let δ be a fuzzy supra residual set in (X,T^*). Then by proposition 4.7, there exists a fuzzy supra strongly residual set η in (X,T^*) such that $\eta \leq \delta$. Then $cl^*(\eta) \leq cl^*(\delta)$ in (X,T^*). By hypothesis $cl^*(\eta) = 1$ in (X,T^*) and thus $1 \leq cl^*(\delta)$. That is, $cl^*(\delta) = 1$ in (X,T^*). Hence, by theorem 2.1, (X,T^*) is a fuzzy supra Baire space.

Proposition 5.3.

If $int^*(\mu) = 0$ for each fuzzy supra strongly first category set μ in a fuzzy supra strongly Baire space (X,T^*), then (X,T^*) is a fuzzy supra Baire space.

Proof.

The proof follows from proposition 5.1.

Proposition 5.4.

If $cl^*(\eta) = 1$, for each fuzzy supra strongly residual set η in a fuzzy supra strongly Baire space (X,T^*), then (X,T^*) is a fuzzy supra Baire space.

Proof.

The proof follows from proposition 5.2.

Proposition 5.5.

If $cl^*(\chi) = 1$, for a fuzzy supra first category set in a FSTS (X,T^*), then there exists a fuzzy supra strongly first category set μ in (X,T^*) such that $cl^*(\mu) = 1$.

Proof.

Let χ be a fuzzy supra first category set in (X,T^*). Then by proposition 4.6, there exists a fuzzy supra strongly first category set μ in (X,T^*) such that $\chi \leq \mu$. Then $cl^*(\chi) \leq cl^*(\mu)$, in (X,T^*). By hypothesis, $cl^*(\chi) = 1$ in (X,T^*) and hence $1 \leq cl^*(\mu)$. That is, $cl(\mu) = 1$ in (X,T^*).

Proposition 5.6.

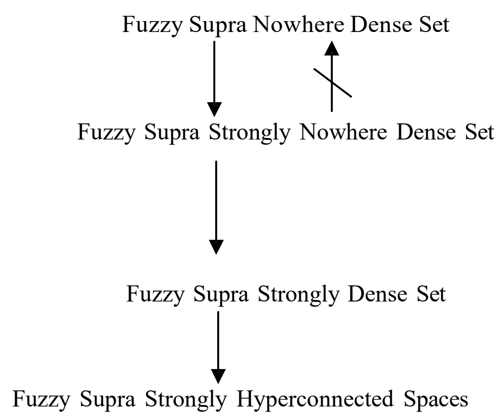
If each fuzzy supra first category set χ is a fuzzy supra dense set in a fuzzy supra Baire space (X,T^*), then (X,T^*) is a fuzzy supra strongly Baire space.

Proof.

Let χ be a fuzzy supra first category set in a fuzzy supra Baire space (X,T^*) such that $cl^*(\chi) = 1$. Then by proposition 5.5, there exists a fuzzy supra strongly first category set μ in (X,T^*) such that $cl^*(\mu) = 1$. Hence, by proposition 4.1, (X,T^*) is a fuzzy supra strongly Baire space.

Remark 5.1

Fuzzy supra nowhere dense sets, Fuzzy supra strongly nowhere dense sets, Fuzzy supra strongly dense sets and Fuzzy supra strongly hyperconnected spaces are summarized as follows.



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