SUPER PRIME GRACEFUL TOTAL GRAPH

Henry and Julie

[1]Research Scholar, Mother Teresa Women's University, Kodaikanal, Dindigul, Tamil Nadu, India. Guest Lecturer, Department of Mathematics, M.Prasad Government Arts College for Women, Dindigul, Tamil Nadu, India.
[2] Associate Professor and Head, Department of Mathematics, Aravind Government Arts College for Women, Dindigul, Tamil Nadu, India.

Abstract

A total graph is called super prime graceful if it admits a labeling of the notes by different positive integers such that any two neighborhood nodes have mutually prime and the arcs by the distinct absolute difference between its endpoints. In this paper we describe the super prime graceful total labeling and show that disconnected graph $G \cup \{w\}$ where $G \cong S_n + K_1$ and bistar $B_{n,n}$ are super prime graceful total graphs.

Keywords : Prime labeling, Graceful labeling, Total labeling, Super prime graceful total labeling.

AMS Subject Classification: 05C78

1. Introduction

Graph theory has obvious utility in its applications in Science. However, when viewed apart from these applications, it yields beautiful mathematical gems, and can be used as a lens to study other area of mathematics. We use graph theory and specifically graph labeling as a lens to study total labeling.

Mathematically in graph hypothesis, a graph labeling is the allocation of labels (commonly represented by an integer) to the graph nodes or graph arcs or both. There are several types of labeling. Motivated by the concepts of super graceful and prime labelings, we define a super prime graceful total labeling. In this paper we derive a disconnected graph $G \cup \{w\}$ where $G \cong K_{1,n} + K_1$ and bistar $B_{n,n}$ are super prime graceful total graph.

2. Review of literature

The notion of graph labeling (i.e., β-valuation) was introduced by Rosa [25]. Later, β-valuation was called graceful by Golomb [10]. Recently, Aljohani and Daoud [2], Makadia et. al. [17] and Velmurugan and Ramachandran [28] developed the different ideas of graceful labeling. The various types of labeling are investigated by Gallian [8]. One of graph labeling is prime labeling that was originated by Entringer [6] and was introduced by Tout et al. [27]

in 1982. Fu and Hung [7], Haxell et al. [11], Deretsky et al. [5] and Lee et al. [16] developed the prime labeling of graphs. For recent result of prime labeling we refer to Abughazaleh and Abughneim [1], Kansagara and Patel [12], Kavitha and Vimala [13] and Ramachandran and Gnanaseelan [23]. Perumal et al. [20] defined super graceful labeling and improved by Perumal et al. [21] and Lau et al. [15]. Ravi and Kala [24] introduced the concept of total prime labeling. Meena and Ezhil [18] have investigate the total prime labeling of some graph. Gnanajothi and Suganya [9] expanded on this work and introduced highly total prime labeling. Kavitha [14] developed the highly total prime labeling for some duplicate graph.

3. Preliminaries

A graph is an ordered pair $G = (V, E)$, where V is the set of all nodes of G, which is non empty and E is the set of all arcs of G. Two nodes x, y in a graph G are said to be neighborhood in G if xy is an arc of G. A simple graph is a graph without loops and multiple arcs. A graph is a connected graph if, for each pair of nodes, there exists at least one single path which joins them. A disconnected graph is a graph that is not connected. There is at least one pair of nodes that have no path connecting them. A finite graph is a graph in which the node set and the arc set are finite sets. An undirected graph is graph, i.e., a set of objects (called nodes or arcs) that are connected together, where all the arcs are bidirectional. The degree of a node u is the number of arcs incident to u. The star is a graph consisting of one node of degree n (called the center) and n nodes of degree 1. A star graph with $n+1$ nodes and n arcs is termed as S_n.

Next, we define a basic definition of labeling of graphs. Graph labeling is an injective mapping from elements of a graph (can be nodes, arcs, faces, or a combination) to a set of numbers (usually positive integers). If the domain of the mapping is the set of nodes then the labeling is called a node labeling. If the domain of the mapping is the set of arcs then the labeling is called an arc labeling. if the domain is $V(G) \cup E(G)$ then the labeling is called total labeling.

It is useful to recall some useful definitions of graph theory to make this article self-contained.

Definition 3.1 (Rosa [25]) Let $G = (V, E)$ be a graph. Let f be a function. A graph G is β-valuation if each node u is assigned a non negative integer $f(u)$ and each arc uv is assigned the absolute value of the difference of the numbers at its endpoints, that is $|f(u) - f(v)|$.

Definition 3.2 (Golomb [10]) Let $G=(V,E)$ be a graph. Let f be a function. Then a graph G is called graceful if

i. The nodes are labelled with distinct integers i.e., f is an injection.

ii. The largest value of the node labels is equal to the number of arcs, i.e., $\max_v f(u) = |E| = e$ (or $f(u) \in \{0, 1, 2, \ldots, e\}$).

iii. All the edges of G are distinctly labelled with the integers from 1 to e, i.e., $\{|f(u)-f(v)| : uv \in E\} = \{1, 2, \ldots, e\}$.

Definition 3.3 (Tout et al. [27]) Let $G=(V,E)$ be a graph. Let g be a function. Then a graph G is called prime if

i. The nodes are labelled with distinct integers (i.e., g is a one-to-one correspondance function).

ii. The largest value of the node labels is equal to the number of nodes, i.e., $\max_v g(u) = |V| = v$ (or $g(u) \in \{1, 2, \ldots, v\}$).

iii. Any two neighborhood nodes have mutually prime labels (or equivalently: $\gcd\{g(x), g(y)\} = 1$).

Definition 3.4 Let $G=(V,E)$ be a graph. Let h be a function from $V \cup E$ into the integers. Then the graph G is called total graph if

i. The nodes and arcs are labelled with distinct integers

ii. The range of h is the set $\{1, 2, \ldots, (|V|+|E|)\}$. Moreover, hence h is necessarily a one-to-one correspondance function.

Definition 3.5 (Perumal et al. [20]) Let $G=(V,E)$ be a graph. Let h be a total labeling. Then h is called a super graceful labeling if

i. Each node u is assigned a non negative integer $h(u)$ and each arc uv is assigned the absolute value of the difference of the numbers at its endpoints, that is $|h(u)-h(v)|$.

ii. The largest value of the node labels is $v+e$, that is $\max_v f(u) = |V| + |E| = v + e$.

Definition 3.6 (Bondy and Murthy [4]) A simple graph G is said to be complete if every pair of distinct nodes of G are neighborhood in G. It is denoted by K_n.

Definition 3.7 (Balakrishnan and Ranganathan [3]) Let $G_1 = (V_1; E_1)$ and $G_2 = (V_2; E_2)$ be two graphs. Then union of G_1 and G_2 is denoted by $G_1 \cup G_2$ is the graphs whose node set is

$V_1 \cup V_2$ and arc set is $E_1 \cup E_2$. When G_1 and G_2 are node disjoint $G_1 \cup G_2$ is called sum of G_1 and G_2 and it is denoted by $G_1 + G_2$.

Definition 3.8 (Ragukumara and Sethuraman [22]) The binomial tree B_0 consists of a single vertex. The binomial tree B_k is an ordered tree defined recursively. The binomial tree B_k consists of two binomial trees B_{k-1} that are linked together: the root of one is the leftmost child of the root of the other. It is noted that there are $2k$ nodes in the binomial tree B_k.

Definition 3.9 (Nivedha and Yamini [19]) For $n \geq 2$, the bistar is a connected graph with $V\left(B_{n,n}\right) = \left\{u, v, u_i, v_i : 1 \leq i \leq n\right\}$ and $E\left(B_{n,n}\right) = \left\{uv, uu_i, vv_i : 1 \leq i \leq n\right\}$ and is denoted by $B_{n,n}$.

Definition 3.10 (Singh [26]) In a rooted tree, vertex v_i is said to be at level l_i if v_i is at a distance l_i from the root and the root is at level 0. An Olive tree is a rooted tree consisting of k branches, where the i-th branch is a path of length i.

4. Super prime graceful total graph

We combine the super graceful labeling and prime labeling which is a new labeling defined in this paper called super prime graceful total labeling.

Definition 4.1 Let $G = (V, E)$ be a graph with v nodes and e arcs. Let $h : V \cup E \rightarrow \left\{1, 2, 3, ..., v + e\right\}$ be a function. Then the function h is called a super prime graceful labeling if

i. Each node u is assigned a non negative integer $h(u)$ and each arc uv is assigned the absolute value of the difference of the numbers at its endpoints, that is $| h(u) - h(v) |$.

ii. The label of each pair of neighborhood nodes are mutually prime (or for each arc $e = xy$, $\gcd\left\{f\left(x\right), f\left(y\right)\right\} = 1$).

If a graph G is not super graceful, then G is not a super prime graceful total graph. The complete graph K_n $\left(n \leq 3\right)$ is super graceful graph (See Lau et al. [15]). But this graph is not a super prime graceful total graph. So, if G admits a super graceful labeling, then G need not admit a super prime graceful total labeling.

Example 4.1 Super prime graceful total labeling of Olive tree for $n = 1, 2, 3, 4, 5, 6$ is shown in Figure 1, Figure 2 and Figure 3.

Example 4.2 Super prime graceful total labeling of Binomial tree for $k = 0, 1, 2, 3, 4$ is shown in Figure 4.

The following theorem shows that the disconnected graph $G \cup \{w\}$ admits a super prime graceful total labeling where $G \cong S_n + K_1$ is a connected graph whose node set is $V(G) = \{u, v\} \cup \{w_i / 1 \le i \le n\}$ and arc set is $E(G) = \{uv\} \cup \{uw_i, vw_i / 1 \le i \le n\}$.

Theorem 4.1. For any integer n, $G \cup \{w\}$ is super prime graceful total graph where $G \cong S_n + K_1$.

Proof. Define a total labeling h from $V \cup E$ to $\{1, 2, \ldots, 3n + 4 = m\}$ as follows:

$$h(x) = \begin{cases} 1 & \text{if } x = u, \\ m - 2 & \text{if } x = w. \end{cases}$$

The case of prime m:

$$h(x) = \begin{cases} m & \text{if } x = v, \\ 3i & \text{if } x = w_i \text{ for } 1 \le i \le n, \\ m - 1 & \text{if } x = uv, \\ 3i - 1 & \text{if } x = uw_i \text{ for } 1 \le i \le n, \\ m - 3i & \text{if } x = vw_i \text{ for } 1 \le i \le n. \end{cases}$$

The case of composite m:

If there is a largest prime $p < m$ such that $p = 3k + 1$ for some k is a positive integer, then m is congruent to $3i$ modulo p where i is a positive integer.

$$h(x)=\begin{cases} p & \text{if } x=v,\\ 3(j+i) & \text{if } x=w_j \text{ for } 1\le j\le n-(3i-1) \text{ and } n\ne 2,\\ 3(n-2i+1)+2j & \text{if } x=w_{n-3i+j+1} \text{ for } 1\le j\le \left\lfloor \frac{3i-1}{2}\right\rfloor,\\ 3(n-2i+1)+2(t+j-1)+5 & \text{if } x=w_{n-3i+t+j+1} \text{ for } 1\le j\le \left\lceil \frac{3i-1}{2}\right\rceil \text{ and } t=\left\lfloor \frac{3i-1}{2}\right\rfloor,\\ p-1 & \text{if } x=uv,\\ 3(j+i)-1 & \text{if } x=uw_j \text{ for } 1\le j\le n-(3i-1) \text{ and } n\ne 2,\\ \left|3(j+i)-p\right| & \text{if } x=vw_j \text{ for } 1\le j\le n-(3i-1) \text{ and } n\ne 2,\\ 3(n-2i+1)+2j-1 & \text{if } x=uw_{n-3i+j+1} \text{ for } 1\le j\le \left\lfloor \frac{3i-1}{2}\right\rfloor,\\ \left|3(n-2i+1)+2j-p\right| & \text{if } x=vw_{n-3i+j+1} \text{ for } 1\le j\le \left\lfloor \frac{3i-1}{2}\right\rfloor,\\ 3(n-2i+1)+2(t+j-1)+4 & \text{if } x=uw_{n-3i+t+j+1} \text{ for } 1\le j\le \left\lceil \frac{3i-1}{2}\right\rceil \text{ and } t=\left\lfloor \frac{3i-1}{2}\right\rfloor,\\ \left|3(n-2i+1)+2(t+j-1)+5-p\right| & \text{if } x=vw_{n-3i+t+j+1} \text{ for } 1\le j\le \left\lceil \frac{3i-1}{2}\right\rceil \text{ and } t=\left\lfloor \frac{3i-1}{2}\right\rfloor. \end{cases}$$

Clearly h is a total labeling from $V\cup E$ to $\{1, 2, \ldots, 3n+4\}$. It can be verified that h is one-to-one correspondance function with $h(xy)=|h(x)-h(y)|$ for every arc $xy\in E$. Also, it can be checked that for each neighborhood nodes receive mutually prime labels, i.e., $\gcd(h(x),h(y))=1$ for every $x, y\in V$ and $xy\in E$. Hence, the required graph is super prime graceful total graph.

Our next result shows that the bistar $B_{n,n}$ admits super prime graceful total labeling.

Theorem 4.2. For any integer $n\ge 1$, the bistar $B_{n,n}$ is super prime graceful total graph.

Proof. Define a total labeling h as follows:

$$h(x)=\begin{cases} 1 & \text{if } x=v,\\ 3 & \text{if } x=u \text{ for } n\le 2,\\ 5 & \text{if } x=u \text{ for } n>2,\\ 2 & \text{if } x=uv \text{ for } n\le 2,\\ 4 & \text{if } x=uv \text{ for } n>2. \end{cases}$$

$$h(x)=\begin{cases}2(2i+1)+1 & if\ x=u_i\ \ for\ 1\le i\le n\ and\ n\le 2,\\ 2(2i+1) & if\ x=v_i\ \ for\ 1\le i\le n\ and\ n\le 2,\\ 4i & if\ x=uu_i\ \ for\ 1\le i\le n\ and\ n\le 2,\\ 4i+1 & if\ x=vv_i\ \ for\ 1\le i\le n\ and\ n\le 2.\end{cases}$$

To label the nodes and arcs of two copies of star of $B_{n,n}$, we consider three cases depending on the values of n, $n>2$.

The case of $3|n$:

If $n=3k_1\ \forall\ k_1\in\square$, then

$$h(x)=\begin{cases}10i+j & if\ x=u_{3(i-1)+j}\ \ for\ 1\le i\le k_1\ and\ j=1,2,3,\\ 5(2i-1)+j & if\ x=uu_{3(i-1)+j}\ \ for\ 1\le i\le k_1\ and\ j=1,2,3,\\ 5(i+1) & if\ x=v_{i+1}\ \ for\ 1\le i\le 2k_1,\\ 5i+4 & if\ x=vv_{i+1}\ \ for\ 1\le i\le 2k_1,\\ 5(2k_1+1)+2i & if\ x=v_{2k_1+i+1}\ \ for\ 1\le i\le n-(2k_1+1)\ and\ k>1,\\ 2(5k_1+i+2) & if\ x=vv_{2k_1+i+1}\ \ for\ 1\le i\le n-(2k_1+1)\ and\ k>1.\end{cases}$$

The case of $3|n-1$:

We take $n=3k_2+1\ \forall\ k_2\in\square$. Label the nodes and arcs of $B_{n,n}$ in the following way,

Let $k_2=1$. Then

$$h(x)=\begin{cases}7,8,11,17 & if\ according\ to\ x=u_i\ \ for\ i=1,2,3,4,\\ 10,14,16,19 & if\ according\ to\ x=v_i\ \ for\ i=1,2,3,4,\\ 2,3,6,12 & if\ according\ to\ x=uu_i\ \ for\ i=1,2,3,4,\\ 9,13,15,18 & if\ according\ to\ x=vv_i\ \ for\ i=1,2,3,4.\end{cases}$$

Let $k_2>1$. Then

$$h(x)=\begin{cases}10i+j & \text{if } x=u_{3(i-1)+j} \text{ for } 1\le i\le k_2 \text{ and } j=1,2,3,\\ 10k_2+11 & \text{if } x=u_n,\\ 5(2i-1)+j & \text{if } x=uu_{3(i-1)+j} \text{ for } 1\le i\le k_2 \text{ and } j=1,2,3,\\ 2(5k_2+3) & \text{if } x=uu_n,\\ 5(i+1) & \text{if } x=v_{i+1} \text{ for } 1\le i\le 2k_2,\\ 5i+4 & \text{if } x=vv_{i+1} \text{ for } 1\le i\le 2k_2,\\ 2(5k_2+i+3) & \text{if } x=v_{2k_2+i+1} \text{ for } i=1,2,\\ 5(2k_2+1)+2i & \text{if } x=vv_{2k_2+i+1} \text{ for } i=1,2,\\ 10k_2+13 & \text{if } x=v_{2(k_2+2)} \text{ for } k_2>2,\\ 2(5k_2+6) & \text{if } x=vv_{2(k_2+2)} \text{ for } k_2>2,\\ 5(2k_2+3) & \text{if } x=v_{2(k_2+2)+1} \text{ for } k_2>3,\\ 2(5k_2+7) & \text{if } x=vv_{2(k_2+2)+1} \text{ for } k_2>3,\\ 5(2k_2+3)+2i & \text{if } x=v_{2k_2+i+5} \text{ for } 1\le i\le k_2-4 \text{ and } k_2>4,\\ 2(5k_2+i+7) & \text{if } x=vv_{2k_2+i+5} \text{ for } 1\le i\le k_2-4 \text{ and } k_2>4.\end{cases}$$

The case of $3|n-2$:

We take $n=3k_3+2 \ \forall\ k_3\in □$. Label the nodes and arcs of $B_{n,n}$ in the following way,

Let $k_3=1$. Then

$$h(x)=\begin{cases}7,8,11,17,23 & \text{if according to } x=u_i \text{ for } i=1,2,3,4,5,\\ 10,14,16,20,22 & \text{if according to } x=v_i \text{ for } i=1,2,3,4,5,\\ 2,3,6,12,18 & \text{if according to } x=uu_i \text{ for } i=1,2,3,4,5,\\ 9,13,15,19,21 & \text{if according to } x=vv_i \text{ for } i=1,2,3,4,5.\end{cases}$$

Let $k_3=2$. Then

$$h(x)=\begin{cases}7,8,11,17,18,19,27,33 & \text{if according to } x=u_i \text{ for } i=1,2,3,4,5,6,7,8,\\ 10,16,21,24,26,30,32,35 & \text{if according to } x=v_i \text{ for } i=1,2,3,4,5,6,7,8,\\ 2,3,6,12,13,14,22,28 & \text{if according to } x=uu_i \text{ for } i=1,2,3,4,5,6,7,8,\\ 9,15,20,23,25,29,31,34 & \text{if according to } x=vv_i \text{ for } i=1,2,3,4,5,6,7,8.\end{cases}$$

Let $k_3>2$. Then

$$h(x)=\begin{cases} 10i+j & \text{if } x=u_{3(i-1)+j} \text{ for } 1\le i\le k_3 \text{ and } j=1,2,3,\\ 10k_3+11 & \text{if } x=u_{n-1},\\ 10k_3+17 & \text{if } x=u_n,\\ 5(2i-1)+j & \text{if } x=uu_{3(i-1)+j} \text{ for } 1\le i\le k_3 \text{ and } j=1,2,3,\\ 2(5k_3+3) & \text{if } x=uu_{n-1},\\ 2(5k_3+6) & \text{if } x=uu_n,\\ 5(i+1) & \text{if } x=v_{i+1} \text{ for } 1\le i\le 2k_3,\\ 5i+4 & \text{if } x=vv_{i+1} \text{ for } 1\le i\le 2k_3,\\ 2(5k_3+i+3) & \text{if } x=v_{2k_3+i+1} \text{ for } i=1,2,\\ 5(2k_3+1)+2i & \text{if } x=vv_{2k_3+i+1} \text{ for } i=1,2,\\ 2(5k_3+i+6) & \text{if } x=v_{2k_3+i+3} \text{ for } i=1,2,\\ 10k_3+2i+11 & \text{if } x=vv_{2k_3+i+3} \text{ for } i=1,2,\\ 10k_3+2i+17 & \text{if } x=v_{2k_3+i+5} \text{ for } 1\le i\le k_3-3 \text{ and } k_3>3,\\ 2(5k_3+i+8) & \text{if } x=vv_{2k_3+i+5} \text{ for } 1\le i\le k_3-3 \text{ and } k_3>3. \end{cases}$$

Clearly h is a total labeling from $V(B_{n,n}) \cup E(B_{n,n})$ to $\{1, 2, \ldots, 4n+3\}$. It can be verified that h is one-to-one correspondance function with $h(xy) = |h(x) - h(y)|$ for every arc $xy \in E(B_{n,n})$. Also, it is straightforward to verify that all neighborhood nodes receive mutually prime labels, i.e., $\gcd(h(x), h(y)) = 1$ for every $x, y \in V(B_{n,n})$ and $xy \in E(B_{n,n})$. Hence, $B_{n,n}$ is super prime graceful total graph.

Conclusion

In this paper, we have studied about the super prime graceful total labeling. It is proved that $G \cup \{w\}$ where $G \cong S_n + K_1$ and $B_{n,n}$ admit super prime graceful total labeling. This work may be extended to (1) super prime graceful total labeling for binomial tree of all order, (2) super prime graceful total labeling for Olive tree of all order and (3) super prime graceful total labeling of fan related graphs.

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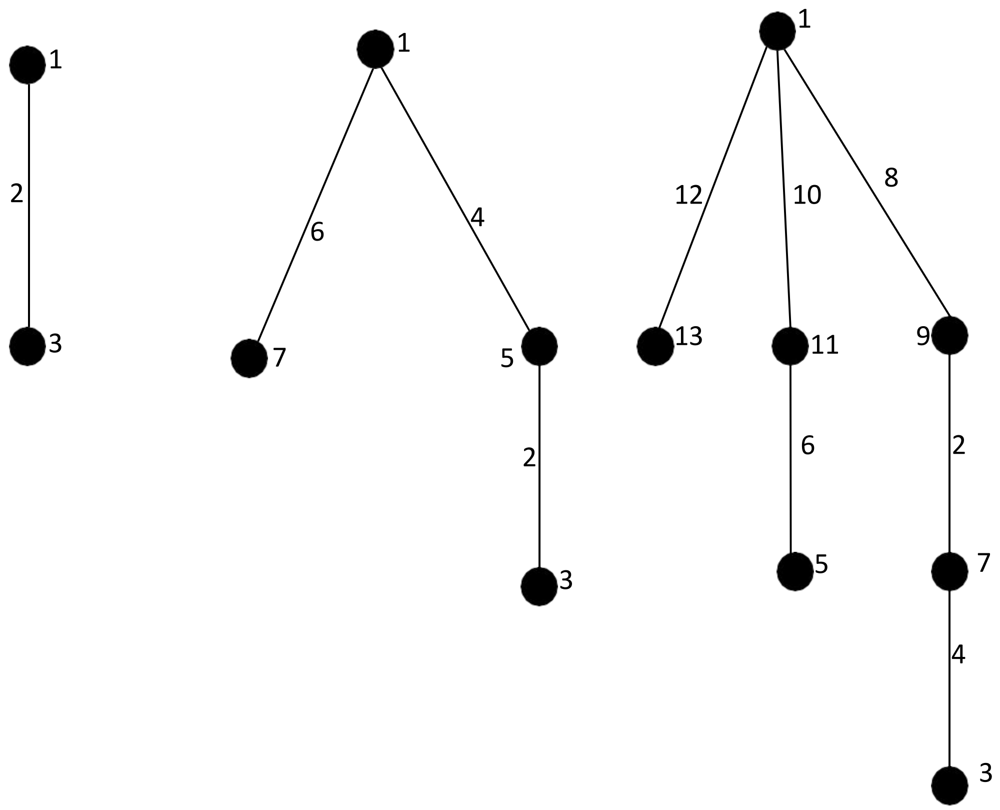

Figure 1 Super prime graceful total labeling of Olive tree for $n = 1, 2, 3$.

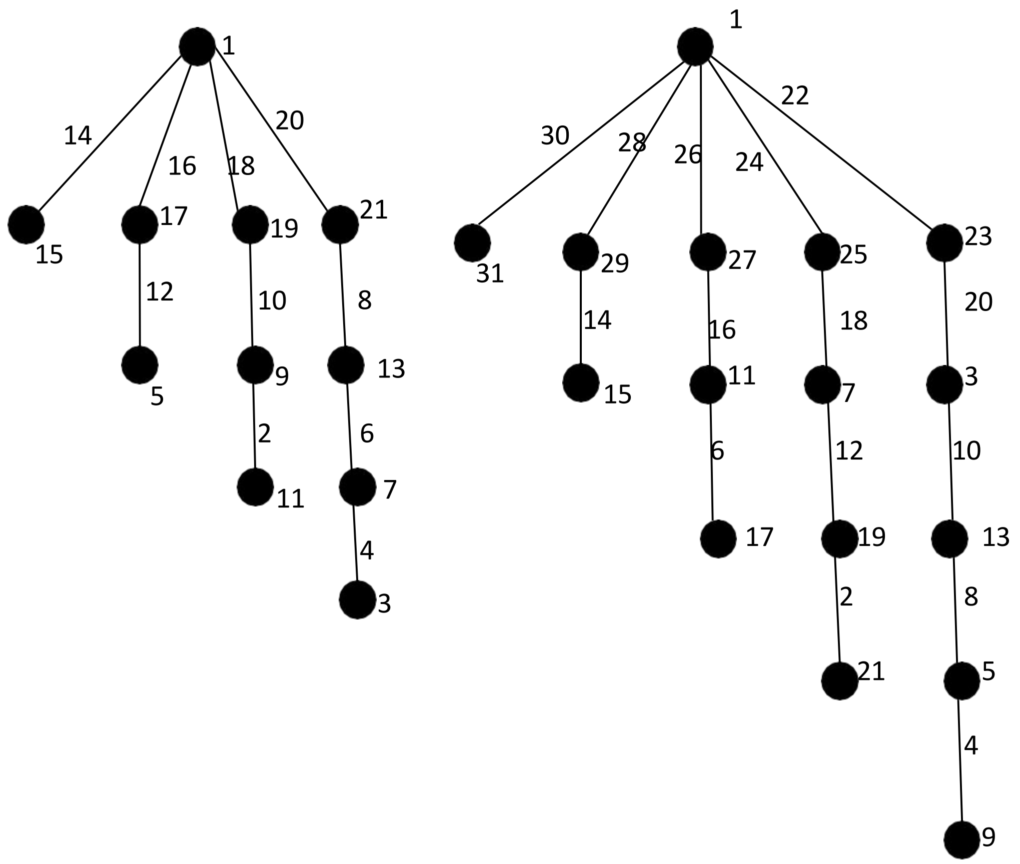

Figure 2 Super prime graceful total labeling of Olive tree for $n = 4, 5$.

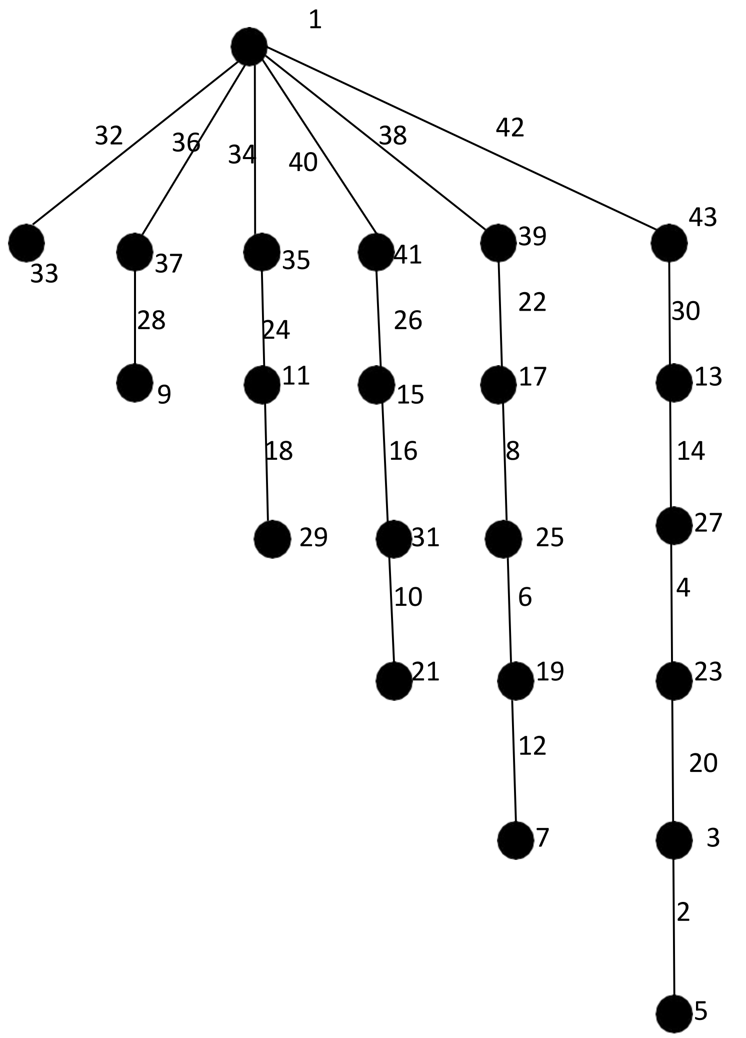

Figure 3 Super prime graceful total labeling of Olive tree for $n = 6$.

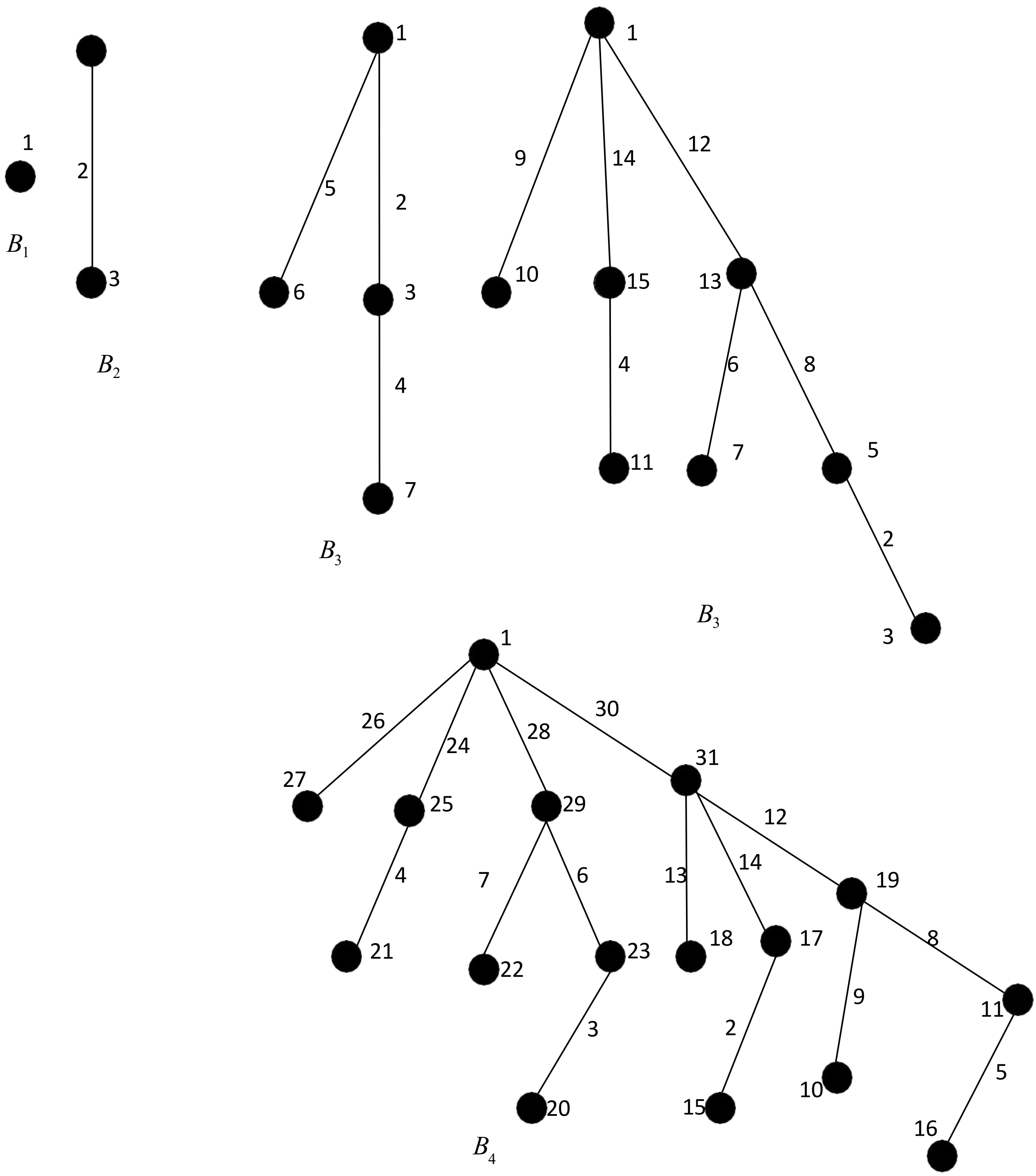

Figure 4 Super prime graceful total labeling of Binomial tree for $k = 0,1,2,3,4$.