PAIRWISE FUZZY F′- SPACES

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ABSTRACT

In this paper, the concept of pairwise fuzzy F′-Spaces is introduced and studied. Several characterizations of pairwise fuzzy F′-Spaces are established.

Keywords: Pairwise fuzzy dense set, pairwise fuzzy G_δ -set, pairwise fuzzy F_σ -set, pairwise fuzzy σ -boundary set, pairwise fuzzy pseudo open set, pairwise fuzzy globally disconnected space.

1. INTRODUCTION

In order to deal with uncertainties, the notions of fuzzy sets and fuzzy set operations were introduced by L.A.Zadeh [18] in his classical paper in the year 1965, describing fuzziness mathematically for the first time. In 1968, C.L.Chang [4] defined fuzzy topological spaces by using fuzzy sets. In 1989, A.Kandil [2] introduced and studied fuzzy bitopological spaces as a natural generalization of fuzzy topological spaces.

In classical topology, F′ - Spaces were introduced by Leonard Gillman and Melvin Henrilsen [8], in which disjoint cozero-sets have disjoint closures. Motivated on these lines, the concept of pairwise fuzzy F′-space is introduced in this paper. Several characterizations of pairwise fuzzy F′-Spaces are established.

2. PRELIMINARIES

In a fuzzy bitopological space (Kandil, 1989) we mean an ordered triple (X,T_1,T_2), where T_1 and T_2 are fuzzy topologies on the non-empty set X. Let X be a non-empty set and I be the unit interval [0,1]. A fuzzy set λ in X is a mapping from X into I.

Definition 2.1. [10] A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy open set if $\lambda \in T_i$ $(i = 1, 2)$. The complement of pairwise fuzzy open set in (X,T_1,T_2) is called a pairwise fuzzy closed set.

Lemma 2.1. [2] For a fuzzy set λ in a fuzzy topological space X,

(i) $1 - \text{int}(\lambda) = \text{cl}(1 - \lambda)$, (ii) $1 - \text{cl}(\lambda) = \text{int}(1 - \lambda)$.

Definition 2.2.[10] A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy G_δ-set if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are pairwise fuzzy open sets in (X,T_1,T_2).

Definition 2.3. [10] A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy F_σ-set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are pairwise fuzzy closed sets in (X,T_1,T_2).

Definition 2.4. [9] A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy dense set if $cl_{T1}\, cl_{T2}(\lambda) = cl_{T2}\, cl_{T1}(\lambda) = 1$, in (X,T_1,T_2).

Definition 2.5 [12]. A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy nowhere dense set if $int_{T1}\, cl_{T2}(\lambda) = int_{T2}\, cl_{T1}(\lambda) = 0$, in (X,T_1,T_2).

Definition 2.6. [10] A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy σ-nowhere dense set if λ is a pairwise fuzzy F_σ-set in (X,T_1,T_2) such that $int_{T1} int_{T2}(\lambda) = int_{T2}\, int_{T1}(\lambda) = 0$.

Definition 2.7.[12] Let (X,T_1,T_2) be a fuzzy bitopological space. A fuzzy set λ in (X,T_1,T_2) is called a pairwise fuzzy first category set if $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X,T_1,T_2). Any other fuzzy set in (X,T_1,T_2) is said to be a pairwise fuzzy second category set in (X,T_1,T_2).

Definition 2.8. [12] If λ is a pairwise fuzzy first category set in a fuzzy bitopological space (X,T_1,T_2), then the fuzzy set $1 - \lambda$ is called a pairwise fuzzy residual set in (X,T_1,T_2).

Definition 2.9 A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a

(i) Pairwise fuzzy semi-open set if $\lambda \leq cl_{T_i}\, int_{T_j}(\lambda)$ ($i \neq j$, and $i,j=1,2$) [7].

(ii) Pairwise fuzzy semi-closed set if $int_{T_i}\, cl_{T_j}(\lambda) \leq \lambda$ ($i \neq j$, and $i,j=1,2$) [7].

(iii) Pairwise fuzzy pre-open set if $\lambda \leq int_{Ti}\, cl_{Tj}(\lambda)$ ($i \neq j$, and $i,j=1,2$) [7].

(iv) Pairwise fuzzy pre-closed set if $cl_{Ti}\, int_{Tj}(\lambda) \leq \lambda$ ($i \neq j$, and $i,j=1,2$) [7].

Definition 2.10 A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called

(i) Pairwise fuzzy regular open set if $int_{T_1}\, cl_{T_2}(\lambda) = \lambda = int_{T_2}\, cl_{T_1}(\lambda)$ [3].

(ii) Pairwise fuzzy regular closed set if $cl_{T_1}\, int_{T_2}(\lambda) = \lambda = cl_{T_2}\, int_{T_1}(\lambda)$ [3].

Definition 2.11. [13] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy submaximal space if each pairwise fuzzy dense set in (X, T_1, T_2), is a pairwise fuzzy open set in (X, T_1, T_2). That is., if λ is a pairwise fuzzy dense set in a fuzzy bitopological space (X, T_1, T_2), then $\lambda \in T_i (i = 1, 2)$.

Definition 2.12.[17] A fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy globally disconnected space if each pairwise fuzzy semi-open set is a pairwise fuzzy open set in (X,T_1,T_2). That is, if $\lambda \leq cl_{T_i}\, int_{T_j}(\lambda)$ ($i \neq j$ and $i,j = 1,2$), for a fuzzy set λ defined on X in a fuzzy bitopological space (X,T_1,T_2), then $\lambda \in T_i$ ($i = 1, 2$).

Theorem 2.1 [13]**:** If λ is a pairwise fuzzy pseudo-open set in a pairwise fuzzy submaximal space (X,T_1,T_2), then $\lambda = \mu \vee \delta$, where μ is a pairwise fuzzy open set and λ is a pairwise fuzzy F_σ-set in (X,T_1,T_2).

Theorem 2.2 [15]: If λ is a pairwise fuzzy σ-boundary set in a pairwise fuzzy topological space (X,T_1,T_2), λ is a pairwise fuzzy F_σ-set in (X,T_1,T_2).

Theorem 2.3 [17]: If λ is a pairwise fuzzy first category set in a pairwise fuzzy globally disconnected space (X,T_1,T_2), then λ is a pairwise fuzzy F_σ-set in (X,T_1,T_2).

Theorem 2.4 [11]:

(a). The closure of a pairwise fuzzy open set is a pairwise fuzzy regular closed set, and

(b). the interior of a pairwise fuzzy closed set is a pairwise fuzzy regular open set.

3. PAIRWISE FUZZY F′ - SPACES

Definition 3.1: A fuzzy bitopologicalspace (X, T_1, T_2) is called a pairwise fuzzy F′- space if $\lambda \le 1 - \mu$, where λ and μ are pairwise fuzzy F_σ -sets in (X,T_1,T_2), then $cl_{T_i}(\lambda) \le 1 - cl_{T_i}(\mu)$, $(i=1,2)$, in (X,T_1,T_2).

Proposition 3.1: If $\lambda + \mu \le 1$, for any two pairwise fuzzy F_σ -sets λ and μ in a pairwise fuzzy F′- space (X,T_1,T_2), then $cl_{T_i}(\lambda) + cl_{T_i}(\mu) \le 1$, in (X,T_1,T_2).

Proof.

Suppose that $\lambda + \mu \le 1$, where λ and μ are pairwise fuzzy F_σ -sets in a pairwise fuzzy F′-space (X,T_1,T_2). Then $\lambda \le 1 - \mu$, in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy F′- space, $cl_{T_i}(\lambda) \le 1 - cl_{T_i}(\mu)$ in (X,T_1,T_2) and then $cl_{T_i}(\lambda) + cl_{T_i}(\mu) \le 1$, in (X,T_1,T_2).

Proposition 3.2: If δ and η are any two pairwise fuzzy G_δ -sets such that $1 - \eta \le \delta$ in a pairwise fuzzy F′ -space, then

(i). $1 - int_{T_i}(\eta) \le int_{T_i}(\delta)$, in (X,T_1,T_2).
(ii). $1 - \eta \le int_{T_i}(\delta)$, in (X,T_1,T_2).

Proof.

(i). Let δ and η be any two pairwise fuzzy G_δ -sets in (X, T_1, T_2). Such that $1 - \eta \le \delta$. Now $1 - \delta \le 1 - (1 - \eta)$ in (X,T_1,T_2), where $1 - \delta$ and $1 - \eta$ are pairwise fuzzy F_σ - sets in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy F′ - space, $cl_{T_i}(1 - \delta) \le 1 - cl_{T_i}(1 - \eta)$ in (X,T_1,T_2). Then $1 - int_{T_i}(\delta) \le 1 - [1 - int_{T_i}(\eta)]$ and then $1 - int_{T_i}(\delta) \le int_{T_i}(\eta)$,in (X,T_1,T_2). Hence $1 - int_{T_i}(\eta) \le int_{T_i}(\delta)$, in (X,T_1,T_2).

(ii). By (i), $1 - int_{T_i}(\eta) \le int_{T_i}(\delta)$ in (X,T_1,T_2). Now $1 - \eta \le 1 - int_{T_i}(\eta)$ implies that $1 - \eta \le int_{T_i}(\delta)$, in (X,T_1,T_2).

Proposition 3.3: If (X,T_1,T_2) is a pairwise fuzzy F′- space and $\lambda \le 1 - \mu$ for any two non-zero pairwise fuzzy F_σ - sets λ and μ in (X,T_1,T_2), then λ and μ are not pairwise fuzzy dense sets in (X,T_1,T_2).

Proof.

Suppose that $\lambda \le 1 - \mu$ for any two non-zero pairwise fuzzy F_σ - sets λ and μ in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy F′- space, $cl_{T_i}(\lambda) \le 1 - cl_{T_i}(\mu)$ in (X,T_1,T_2). If $cl_{T_i}(\lambda) = 1$ then $1 \le 1 - cl_{T_i}(\mu)$ implies that $cl_{T_i}(\mu) \le 0$. That is., $cl_{T_i}(\mu) = 0$ in (X,T_1,T_2). This will imply that $\mu = 0$ a contradiction to $\mu \ne 0$. If $cl_{T_i}(\mu) = 1$ then $cl_{T_i}(\lambda) \le 1 - 1 = 0$. That is., $cl_{T_i}(\lambda) = 0$ in (X,T_1,T_2). This will imply that $\lambda = 0$, a contradiction to $\lambda \ne 0$. Thus, $cl_{T_i}(\lambda) \ne 1$ and $cl_{T_i}(\mu) \ne 1$ in (X,T_1,T_2). Hence λ and μ are not pairwise fuzzy dense sets in (X,T_1,T_2).

Proposition 3.4: If $\lambda \le 1 - \mu$ for any two pairwise fuzzy F_σ -sets in a pairwise fuzzy F′- space (X,T_1,T_2), then

(i). $\lambda + cl_{T_i}(\mu) \le 1$, in (X,T_1,T_2).

(ii). $\mu + cl_{T_i}(\lambda) \le 1$, in (X,T_1,T_2).

Proof.

Suppose that $\lambda \leq 1-\mu$, where λ and μ are pairwise fuzzy F_σ – sets in (X,T_1, T_2). Since (X,T_1,T_2) is a pairwise fuzzy F′-space, $cl_{Ti}(\lambda) \leq 1 - cl_{Ti}(\mu)$ in (X,T_1,T_2). Now $1 - cl_{Ti}(\mu) \leq 1 - \mu$ and $\lambda \leq cl_{Ti}(\lambda)$ implies that $\lambda \leq cl_{Ti}(\lambda) \leq 1 - cl_{Ti}(\mu) \leq 1 - \mu$ in (X,T_1,T_2). Then

(i). $\lambda + cl_{Ti}(\mu) \leq 1$, in (X,T_1,T_2) and
(ii). $\mu + cl_{Ti}(\lambda) \leq 1$, in (X,T_1,T_2).

Proposition 3.5: If (X,T_1,T_2) is a pairwise fuzzy F′- space in which each pairwise fuzzy open set is a pairwise fuzzy F_σ - set, then for a fuzzy set λ defined on X, $cl_{Ti}int_{Ti}(\lambda) \leq int_{Ti}cl_{Ti}(\lambda)$, in (X,T_1,T_2).

Proof.

Let λ be a fuzzy set in (X,T_1,T_2). Then $int_{Ti}(\lambda) \leq cl_{Ti}(\lambda)$ implies that $int_{Ti}(\lambda) \leq 1 - [1 - cl_{Ti}(\lambda)]$ in (X,T_1,T_2). Since, $int_{Ti}(\lambda)$ and $1 - cl_{Ti}(\lambda)$ are pairwise fuzzy open sets in (X,T_1,T_2), by hypothesis, $int_{Ti}(\lambda)$ and $1 - cl_{Ti}(\lambda)$ are pairwise fuzzy F_σ -sets in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy F′- space, $cl_{Ti}int_{Ti}(\lambda) \leq 1 - cl_{Ti}[1-cl_{Ti}(\lambda)]$ in (X,T_1,T_2). Then, $cl_{Ti}int_{Ti}(\lambda) \leq 1 - [1 - int_{Ti}cl_{Ti}(\lambda)]$ and thus $cl_{Ti}int_{Ti}(\lambda) \leq int_{Ti}cl_{Ti}(\lambda)$, in (X,T_1,T_2).

Proposition 3.6: If (X,T_1,T_2) is a pairwise fuzzy F′- space and $\lambda \leq 1 - \mu$, where λ and μ are pairwise fuzzy σ-nowhere dense sets in (X,T_1,T_2), then $cl_{Ti}(\lambda) \leq 1 - cl_{Ti}(\mu)$, where $int_{Ti}(\lambda) = 0$; $cl_{Ti}(\lambda) \neq 1$ and $int_{Ti}(\mu) = 0$; $cl_{Ti}(\mu) \neq 1$, in (X,T_1,T_2).

Proof.

Suppose that $\lambda \leq 1 - \mu$, for any two pairwise fuzzy σ –nowhere dense sets λ and μ in (X,T_1,T_2). Then λ and μ are pairwise fuzzy F_σ-sets in (X,T_1,T_2). Such that $int_{Ti}(\lambda) = 0$ and $int_{Ti}(\mu) = 0$ in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy F′- space, $cl_{Ti}(\lambda) \leq 1 - cl_{Ti}(\mu)$ in (X,T_1,T_2). By Proposition 3.3, λ and μ are not pairwise fuzzy dense sets in (X,T_1,T_2). Thus $cl_{Ti}(\lambda) \leq 1 - cl_{Ti}(\mu)$, where $int_{Ti}(\lambda) = 0$; $cl_{Ti}(\lambda) \neq 1$ and $int_{Ti}(\mu) = 0$; $cl_{Ti}(\mu) \neq 1$, in (X,T_1,T_2).

Proposition 3.7: If λ is a pairwise fuzzy pseudo-open set in a pairwise fuzzy submaximal space (X,T_1,T_2), then there exists pairwise fuzzy F_σ - set η in (X,T_1,T_2). S uch that $\lambda \geq \eta$, in (X,T_1,T_2).

Proof.

Let λ be a pairwise fuzzy pseudo-open set in a pairwise fuzzy submaximal space (X,T_1,T_2). Then, by Theorem 2.1, $\lambda = \mu \vee \eta$, where $\mu \in T_i$ and η is a pairwise fuzzy F_σ - set in (X,T_1,T_2). This implies that $\lambda \geq \eta$, in (X,T_1,T_2).

Proposition 3.8: If $\lambda \leq 1 - \mu$, for any two pairwise fuzzy pseudo-open sets in a pairwise fuzzy submaximal and pairwise fuzzy F′- space (X,T_1,T_2), then there exists a pairwise fuzzy F_σ - sets α and β in (X,T_1,T_2). Such that

(i). $cl_{Ti}(\alpha) \leq 1 - int_{Ti}(\beta)$, in (X,T_1,T_2).
(ii). $\alpha \leq cl_{Ti}(\lambda) \leq 1 - int_{Ti}(\mu) \leq 1 - \beta$, in (X,T_1,T_2).

Proof.

Suppose that $\lambda \leq 1 - \mu$, where λ and μ are pairwise fuzzy pseudo open sets in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy submaximal space, for the pairwise fuzzy pseudo opensets λ and μ, there exist pairwise fuzzy F_σ - sets α and β in (X,T_1,T_2), such that $\alpha \leq \lambda$ and $\beta \leq \mu$. Then, $\alpha \leq \lambda \leq 1 - \mu \leq 1 - \beta$ and thus $\alpha \leq 1 - \beta$ in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy F′- space, $cl_{Ti}(\alpha) \leq 1-cl_{Ti}(\beta)$ in (X,T_1,T_2). Now $\alpha \leq \lambda$ implies that $cl_{Ti}(\alpha) \leq cl_{Ti}(\lambda)$, in (X,T_1,T_2) and then $\alpha \leq cl_{Ti}(\lambda)$. Also $\lambda \leq 1 - \mu$ implies that $cl_{Ti}(\lambda) \leq cl_{Ti}(1 - \mu)$ and then $cl_{Ti}(\lambda) \leq 1 - int_{Ti}(\mu)$. Now $\beta \leq \mu$ implies that $int_{Ti}(\beta) \leq int_{Ti}(\mu)$ and then $1 - int_{Ti}(\beta) \geq 1 - int_{Ti}(\mu)$. Thus $\alpha \leq cl_{Ti}(\lambda) \leq 1 - int_{Ti}(\mu) \leq 1 - int_{Ti}(\beta) \leq 1 - \beta$, in (X,T_1,T_2). Thus, we have

(i). $cl_{Ti}(\alpha) \leq 1 - int_{Ti}(\beta)$, in (X,T_1,T_2).

(ii). $\alpha \le cl_{Ti}(\lambda) \le 1 - int_{Ti}(\mu) \le 1 - \beta$, in (X,T_1,T_2).

Proposition 3.9: If $\lambda \ge \mu$, where $\lambda(= 0)$ is a pairwise fuzzy G_δ -set and μ is a pairwise fuzzy F_σ -set in a pairwise fuzzy F' -space (X,T_1,T_2), then there exists pairwise fuzzy open set δ in (X,T_1,T_2)such that $cl_{Ti}(\mu) \le \delta \le cl_{Ti}(\lambda)$ in (X,T_1,T_2).

Proof.

Suppose that $\lambda \ge \mu$, where $\lambda(= 0)$ is a pairwise fuzzy G_δ -set and μ is a pairwise fuzzy F_σ -set in (X,T_1,T_2). Then $(1 - \lambda) \le 1 - \mu$ and $1 - \lambda$ and μ are pairwise fuzzy F_σ –sets in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy F'- space, $cl_{Ti}(1 - \lambda) \le 1 - cl_{Ti}(\mu)$ in (X,T_1,T_2) and this implies that $int_{Ti}(\lambda) \ge cl_{Ti}(\mu)$ in (X,T_1,T_2). But $int_{Ti}(\lambda) \le cl_{Ti}(\lambda)$ implies that $cl_{Ti}(\mu) \le int_{Ti}(\lambda) \le cl_{Ti}(\lambda)$ in (X,T_1,T_2). Let $\delta = int_{Ti}(\lambda)$. Then δ is a pairwise fuzzy open set in (X,T_1,T_2) such that $cl_{Ti}(\mu) \le \delta \le cl_{Ti}(\lambda)$ in (X,T_1,T_2).

Proposition 3.10: If $\lambda \le \mu$, where λ is a pairwise fuzzy F_σ -set and $\mu(= 0)$ is a pairwise fuzzy G_δ -set in a pairwise fuzzy F'- space (X,T_1,T_2), then there exists a pairwise fuzzy closed set η in (X,T_1,T_2) such that $int_{Ti}(\lambda) \le \eta \le int_{Ti}(\mu)$.

Proof.

Suppose that $\lambda \le \mu$, where λ is a pairwise fuzzy F_σ -set and μ is a pairwise fuzzy G_δ -set in (X,T_1,T_2). Now, $\lambda \le 1 - (1 - \mu)$, where λ and $1 - \mu$ are pairwise fuzzy F_σ -setsin (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy F' - space, $cl_{Ti}(\lambda) \le 1 - cl_{Ti}(1 - \mu)$ and hence $cl_{Ti}(\lambda) \le int_{Ti}(\mu)$ in (X,T_1,T_2). But $int_{Ti}(\lambda) \le cl_{Ti}(\lambda)$ implies that $int_{Ti}(\lambda) \le cl_{Ti}(\lambda) \le int_{Ti}(\mu)$ in (X,T_1,T_2). Let $\eta = cl_{Ti}(\lambda)$. Thus η is a pairwise fuzzy closed set in (X,T_1,T_2). S uch that, $int_{Ti}(\lambda) \le \eta \le int_{Ti}(\mu)$.

Proposition 3.11: If $\lambda \le 1 - \mu$, where λ and μ are pairwise fuzzy σ -boundary sets in a pairwise fuzzy F'- space (X,T_1,T_2), then $cl_{Ti}(\lambda) \le 1 - cl_{Ti}(\mu)$ in (X,T_1,T_2).

Proof.

Let λ and μ be any two pairwise fuzzy σ -boundary sets in (X,T_1,T_2) such that $\lambda \le 1 - \mu$. Now, by Theorem 2.2, λ and μ are pairwise fuzzy F_σ -sets in (X,T_1,T_2). Since (X,T_1,T_2) is a pairwise fuzzy F'- space, $\lambda \le 1 - \mu$ implies that $cl_{Ti}(\lambda) \le 1 - cl_{Ti}(\mu)$ in (X,T_1,T_2).

Proposition 3.12: If $\lambda \le 1 - \mu$, where λ and μ are pairwise fuzzy first category sets in a pairwise fuzzy globally disconnected and pairwise fuzzy F'- space (X,T_1,T_2), then $cl_{Ti}(\lambda) \le 1 - cl_{Ti}(\mu)$ in (X,T_1,T_2).

Proof.

Let λ and μ be pairwise fuzzy first category sets in a pairwise fuzzy globally disconnected space (X,T_1,T_2). Then, by Theorem 2.3, λ and μ are pairwise fuzzy F_σ -sets in (X,T_1,T_2). Also, since (X,T_1,T_2) is a pairwise fuzzy F'- space, $\lambda \le 1 - \mu$ implies that $cl_{Ti}(\lambda) \le 1 - cl_{Ti}(\mu)$ in (X,T_1,T_2).

Proposition 3.13: If (X,T_1,T_2) is a pairwise fuzzy F'- space in which each pairwise fuzzy open set is a pairwise fuzzy F_σ - set, then for a fuzzy set λ defined on X, there exists a pairwise fuzzy regular open set δ in (X,T_1,T_2) such that $cl_{Ti}int_{Ti}(\lambda) \le \delta$.

Proof.

Let (X,T_1,T_2) be a pairwise fuzzy F'- space. Then, by Proposition 3.5, for a fuzzy set λ defined on X, $cl_{Ti}int_{Ti}(\lambda) \le int_{Ti}cl_{Ti}(\lambda)$, in (X,T_1,T_2). Since the interior of a pairwise fuzzy closed set is a pairwise fuzzy regular open set in a pairwise fuzzy topological space, by Theorem 2.4, $int_{Ti}[cl_{Ti}(\lambda)]$ is a pairwise fuzzy regular open set in (X,T_1,T_2). Let $\delta = int_{Ti}cl_{Ti}(\lambda)$. Thus, there exists a pairwise fuzzy regular open set δ in (X,T_1,T_2) such that $cl_{Ti}int_{Ti}(\lambda) \le \delta$.

Proposition 3.14: If (X,T_1,T_2) is a pairwise fuzzy F'- space in which pairwise fuzzy open sets are pairwise fuzzy F_σ - sets, then there exists a pairwise fuzzy regular closed set η and a pairwise fuzzy regular open set δ in (X,T_1,T_2) such that $\eta \le \delta$.

Proof.

Let (X,T_1,T_2) be a pairwise fuzzy F′- space in which pairwise fuzzy open sets are pairwise fuzzy F_σ - sets. Then, by Proposition 3.5, for a fuzzy set λ defined on X, $cl_{Ti}int_{Ti}(\lambda) \leq int_{Ti}cl_{Ti}(\lambda)$. Let $\eta = cl_{Ti}int_{Ti}(\lambda)$ and $\delta = int_{Ti}cl_{Ti}(\lambda)$. Then, by T heorem 2.4, η is a pairwise fuzzy regular closed set and δ is a pairwise fuzzy regular open set in (X,T_1,T_2). Thus, there exists a pairwise fuzzy regular closed set η and a pairwise fuzzy regular open set δ in (X,T_1,T_2) such that $\eta \leq \delta$.

4. CONCLUSION

In this paper, the concept of pairwise fuzzy F′ - Spaces is introduced and studied. Several characterizations of pairwise fuzzy F′-Spaces are established. Also finding the characterizations of pairwise fuzzy F′-Spaces in a pairwise fuzzy submaximal space, pairwise fuzzy globally disconnected spaces are obtained.

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