ISSN NO: 2249-3034

APPLICATIONS OF $wbg\alpha$-CLOSED SETS AND $wb\alpha g$-CLOSED SETS IN BINARY TOPOLOGICAL SPACE

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ABSTRACT. In this paper, As an application of $wbg\alpha$-closed sets and $wb\alpha g$-closed sets, we will define eleven new spaces namely ${}_{wbga}T_{bc}$-space, ${}_{wbag}T_{bc}$-space, ${}_{wbga}T_{ba}$-space, ${}_{wbag}T_{ba}$-space, ${}_{wbga}T_{bga}$-space, ${}_{wbga}T_{bag}$-space, ${}_{wbag}T_{bag}$-space, ${}_{wbag}T_{bga}$-space, ${}_{wbag}T_{wbga}$-space, ${}_{wbga}T_{bg^*}$ -space, ${}_{wbag}T_{bg^*}$ -space and study some of their properties.

1. Introduction

In 2011, S.Nithyanantha Jothi and P.Thangavelu [11] introduced topology between two sets and also studied some of their properties. For background material, papers [1] to [17] may be perused. In this paper, As an application of $wbg\alpha$-closed sets

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2010 *Mathematics Subject Classification.* 54C05, 54C08, 54C10.

Key words and phrases. $wbg\alpha$-closed sets, $wb\alpha g$-closed sets, ${}_{wbga}T_{bc}$-space and ${}_{wbag}T_{bc}$-space.

and *wbαg*-closed sets, we will define eleven new spaces namely ${}_{wbg\alpha}T_{bc}$-space, ${}_{wb\alpha g}T_{bc}$-space, ${}_{wbg\alpha}T_{b\alpha}$-space, ${}_{wb\alpha g}T_{b\alpha}$-space, ${}_{wbg\alpha}T_{bg\alpha}$-space, ${}_{wbg\alpha}T_{b\alpha g}$-space, ${}_{wb\alpha g}T_{b\alpha g}$-space, ${}_{wb\alpha g}T_{bg\alpha}$-space, ${}_{wb\alpha g}T_{wbg\alpha}$-space, ${}_{wbg\alpha}T_{bg^{\star}}$ -space, ${}_{wb\alpha g}T_{bg^{\star}}$ -space and study some of their properties.

2. Applications of *wbgα*-Closed Sets and *wbαg*-Closed Sets

Definition 2.1. *A binary topological space* $(X, Y, \mathcal{M})$ *is said to be a*

(1) ${}_{wbg\alpha}T_{bc}$-*space if every wbgα-closed set in it is binary closed.*

(2) ${}_{wb\alpha g}T_{bc}$-*space if every wbαg-closed set in it is binary closed.*

(3) ${}_{wbg\alpha}T_{b\alpha}$-*space if every wbgα-closed set in it is bα-closed.*

(4) ${}_{wb\alpha g}T_{b\alpha}$-*space if every wbαg-closed set in it is bα-closed.*

(5) ${}_{wbg\alpha}T_{bg\alpha}$-*space if every wbgα-closed set in it is bgα-closed.*

(6) ${}_{wbg\alpha}T_{b\alpha g}$-*space if every wbgα-closed set in it is bαg-closed.*

(7) ${}_{wb\alpha g}T_{b\alpha g}$-*space if every wbαg-closed set in it is bαg-closed.*

(8) ${}_{wb\alpha g}T_{bg\alpha}$-*space if every wbαg-closed set in it is bgα-closed.*

(9) ${}_{wb\alpha g}T_{wbg\alpha}$-*space if every wbαg-closed set in it is wbgα-closed.*

(10) ${}_{wbg\alpha}T_{bg^{\star}}$-*space if every wbgα-closed set in it is bg* *-*closed.*

(11) ${}_{wb\alpha g}T_{bg^{\star}}$-*space if every wbαg-closed set in it is bg* *-*closed.*

Theorem 2.2. *Every* ${}_{wbg\alpha}T_{bc}$-*space is also a* ${}_{wbg\alpha}T_{bg\alpha}$-*space.*

Proof. Assume that $(X, Y, \mathcal{M})$ is a ${}_{wbg\alpha}T_{bc}$-space. Let (A, B) be a *wbgα*-closed set in (X, Y). Then (A, B) is binary closed, as (X, Y) is ${}_{wbg\alpha}T_{bc}$-space. Since every binary closed set is *bgα*-closed, (A, B) is *bgα*-closed in (X, Y). Thus $(X, Y, \mathcal{M})$ is a ${}_{wbg\alpha}T_{bg\alpha}$-space.

The converse of the above Theorem need not be true as seen from the following example.

Example 2.3. *Let* $X = \{a, b\}$, $Y = \{1, 2\}$ *and* $\mathrm{M} = \{(\phi, \phi), (\phi, \{2\}), (X, \{1\}), (X, Y)\}$. *Then* (X, Y, M) *is a* ${}_{\mathrm{wbg\alpha}}T_{\mathrm{bg\alpha}}$*-space but not a* ${}_{\mathrm{wbg\alpha}}T_{\mathrm{bc}}$*-space, since the subset* $(\{b\}, \{2\})$ *is wbgα-closed but not binary closed in* (X, Y, M).

Theorem 2.4. *Every* ${}_{\mathrm{wb\alpha g}}T_{\mathrm{bc}}$*-space is a* ${}_{\mathrm{wbg\alpha}}T_{\mathrm{bg\alpha}}$*-space (resp.* ${}_{\mathrm{wbg\alpha}}T_{\mathrm{b\alpha g}}$*-space).*

Proof. Assume that (X, Y, M) is a ${}_{\mathrm{wb\alpha g}}T_{\mathrm{bc}}$-space. Let (A, B) be a *wbgα*-closed set in (X, Y). Then (A, B) is *wbαg*-closed. By definition of ${}_{\mathrm{wb\alpha g}}T_{\mathrm{bc}}$-space, (A, B) is binary closed. Since every binary closed set is *bgα*-closed (resp. *bαg*-closed), (A, B) is *bgα*-closed (resp. *bαg*-closed) in (X, Y). Thus (X, Y, M) is a ${}_{\mathrm{wbg\alpha}}T_{\mathrm{bg\alpha}}$-space (resp. ${}_{\mathrm{wbg\alpha}}T_{\mathrm{b\alpha g}}$-space).

The converse of the above Theorem need not be true as seen from the following example.

Example 2.5. *In Example 2.3, then* (X, Y, M) *is both* ${}_{\mathrm{wbg\alpha}}T_{\mathrm{bg\alpha}}$*-space and* ${}_{\mathrm{wbg\alpha}}T_{\mathrm{b\alpha g}}$*-space but not a* ${}_{\mathrm{wb\alpha g}}T_{\mathrm{bc}}$*-space, since the subset* $(\{a\}, \{1\})$ *is wbαg-closed but not binary closed in* (X, Y, M).

Remark 2.6. (1) *Every* ${}_{\mathrm{wbg\alpha}}T_{\mathrm{bc}}$*-space is a* ${}_{\mathrm{wbg\alpha}}T_{\mathrm{b\alpha g}}$*-space.*

(2) *Every* ${}_{\mathrm{wb\alpha g}}T_{\mathrm{bc}}$*-space is a* ${}_{\mathrm{wb\alpha g}}T_{\mathrm{bg\alpha}}$*-space,* ${}_{\mathrm{wb\alpha g}}T_{\mathrm{b\alpha g}}$*-space and* ${}_{\mathrm{wb\alpha g}}T_{\mathrm{wbg\alpha}}$*-space.*

(3) *Every* ${}_{\mathrm{wb\alpha g}}T_{\mathrm{bg\alpha}}$*-space is a* ${}_{\mathrm{wb\alpha g}}T_{\mathrm{wbg\alpha}}$*-space.*

The converse of the above results need not be true as seen from the following examples.

Example 2.7. *Let* $X = \{a, b\}$, $Y = \{1, 2\}$ *and* $\mathrm{M} = \{(\phi, \phi), (\phi, \{1\}), (\{a\}, \{1\}), (\{b\}, \{1\}), (X, \{1\}), (X, Y)\}$. *Then* (X, Y, M) *is a* ${}_{\mathrm{wbg\alpha}}T_{\mathrm{b\alpha g}}$*-space but not a* ${}_{\mathrm{wbg\alpha}}T_{\mathrm{bc}}$*-space, since the subset* $(\{a\}, \phi)$ *is wbgα-closed but not binary closed in* (X, Y, M).

Example 2.8. *Let* $X = \{a, b\}$, $Y = \{1, 2\}$ *and* $\mathrm{M} = \{(\phi, \phi), (\{a\}, \{1\}), (\{b\}, \{2\}), (X, Y)\}$. *Then* (X, Y, M) *is a* ${}_{\mathrm{wb\alpha g}}T_{\mathrm{bg\alpha}}$*-space,* ${}_{\mathrm{wb\alpha g}}T_{\mathrm{\alpha g}}$*-space and* ${}_{\mathrm{wb\alpha g}}T_{\mathrm{wbg\alpha}}$*-space but not a* ${}_{\mathrm{wb\alpha g}}T_{\mathrm{bc}}$*-space, since the subset* $(\{a\}, \{2\})$ *is wbαg-closed but not binary closed in* (X, Y, M).

Example 2.9. *Let* $X = \{a, b\}$, $Y = \{1, 2\}$ *and* $\mathrm{M} = \{(\phi, \phi), (X, \{1\}), (X, Y)\}$. *Then* (X, Y, M) *is a* ${}_{\mathrm{wb\alpha g}}T_{\mathrm{wbg\alpha}}$*-space but not a* ${}_{\mathrm{wb\alpha g}}T_{\mathrm{bg\alpha}}$*-space, since the subset* $(\{b\}, \{1\})$ *is wbαg-closed but not bgα-closed in* (X, Y, M).

Theorem 2.10. *Every* ${}_{\mathrm{wb\alpha g}}T_{\mathrm{bg\alpha}}$*-space is also a* ${}_{\mathrm{wbg\alpha}}T_{\mathrm{b\alpha g}}$*-space.*

Proof. Assume that (X, Y, M) is a ${}_{\mathrm{wb\alpha g}}T_{\mathrm{bg\alpha}}$-space. Let (A, B) be a *wbgα*-closed set in (X, Y). Then (A, B) is *wbαg*-closed. By definition of ${}_{\mathrm{wb\alpha g}}T_{\mathrm{bg\alpha}}$-space, (A, B) is *bgα*-closed. Since every *bgα*-closed set is *bαg*-closed, (A, B) is *bαg*-closed in (X, Y). Thus (X, Y, M) is a ${}_{\mathrm{wbg\alpha}}T_{\mathrm{b\alpha g}}$-space.

The converse of the above Theorem need not be true as seen from the following example.

Example 2.11. *In Example 2.7, then* (X, Y, M) *is a* ${}_{\mathrm{wbg\alpha}}T_{\mathrm{b\alpha g}}$*-space but not a* ${}_{\mathrm{wb\alpha g}}T_{\mathrm{bg\alpha}}$*-space, since the subset* $(\{a\}, Y)$ *is wbαg-closed but not bgα-closed in* (X, Y, M).

Theorem 2.12. *If* (X, Y, M) *is a* ${}_{\mathrm{wbg\alpha}}T_{\mathrm{b\alpha}}$*-space and an binary* T_{α}*-space, then it is* ${}_{\mathrm{wbg\alpha}}T_{\mathrm{bc}}$*-space.*

Proof. Let (A, B) be a *wbgα*-closed set in (X, Y, M). Since (X, Y, M) is a ${}_{\mathrm{wbg\alpha}}T_{\mathrm{b\alpha}}$-space, (A, B) is binary T_{α}-closed in (X, Y, M). Since (X, Y, M) is an binary T_{α}-space, every *bα*-closed set is binary closed and hence (A, B) is binary closed in (X, Y, M). Hence (X, Y, M) is a ${}_{\mathrm{wbg\alpha}}T_{\mathrm{bc}}$-space.

Theorem 2.13. *If* (*X, Y,* M) *is a* ${}_{wbg\alpha}T_{b\alpha}$*-space, then every wbgα-closed set is bgα-closed.*

Proof. Follows from the fact that every *bα*-closed set is *bgα*-closed in (*X, Y,* M).

Theorem 2.14. *Every* ${}_{wbg\alpha}T_{bc}$*-space (resp.* ${}_{wb\alpha g}T_{bc}$*-space) is an binary* T_α*-space.*

Proof. Assume that (*X, Y,* M) is a ${}_{wbg\alpha}T_{bc}$-space (resp. ${}_{wb\alpha g}T_{bc}$-space). Let (*G, H*) be *bα*-closed in (*X, Y,* M). Then (*G, H*) is *wbgα*-closed (resp. *wbαg*-closed), as every *bα*-closed set is *wbgα*-closed (resp. *wbαg*-closed). By definition of ${}_{wbg\alpha}T_{bc}$-space (resp. ${}_{wb\alpha g}T_{bc}$-space), (*G, H*) is binary closed. Hence (*X, Y,* M) is an binary T_α-space.

The converse of the above Theorem need not be true as seen from the following example.

Example 2.15. *Let* $X = \{a, b\}$, $Y = \{1, 2\}$ *and* $M = \{(\phi, \phi), (\{a\}, Y), (X, Y)\}$. *Then* (*X, Y,* M) *is an binary* T_α*-space but not a* ${}_{wbg\alpha}T_{bc}$*-space and* ${}_{wb\alpha g}T_{bc}$*-space, since the subset* $(\{a\}, \phi)$ *is both wbgα-closed and wbαg-closed but not binary closed in* (*X, Y,* M).

Theorem 2.16. *Every* ${}_{wb\alpha g}T_{b\alpha}$*-space is a* ${}_{wbg\alpha}T_{bg\alpha}$*-space.*

Proof. Assume that (*X, Y,* M) is a ${}_{wb\alpha g}T_{b\alpha}$-space. Let (*A, B*) be *wbgα*-closed in (*X, Y*). Then (*A, B*) is *wbαg*-closed. This implies (*A, B*) is *bα*-closed, as (*X, Y,* M) is ${}_{wb\alpha g}T_{b\alpha}$-space. Since every *bα*-closed set is *bgα*-closed and hence (*X, Y,* M) is a ${}_{wbg\alpha}T_{bg\alpha}$-space.

The converse of the above Theorem need not be true as seen from the following example.

Example 2.17. *In Example 2.3, then* (*X, Y,* M) *is a* ${}_{wbg\alpha}T_{bg\alpha}$*-space but not a* ${}_{wb\alpha g}T_{b\alpha}$*-space, since the subset* $(\{b\}, \{1\})$ *is wbαg-closed but not bα-closed in* (*X, Y,* M).

Theorem 2.18. *If* (X, Y, M) *is a* ${}_{wbg\alpha}T_{b\alpha}$*-space, then* $wbg\alpha\text{-}cl(E, F) = b\alpha\text{-}cl(E, F)$ *for each subset* (E, F) *of* (X, Y, M).

Proof. We know that, every $b\alpha$-closed set is $wbg\alpha$-closed in (X, Y, M). Since (X, Y, M) is a ${}_{wbg\alpha}T_{b\alpha}$-space, every $wbg\alpha$-closed set is $b\alpha$-closed. Hence $WBG\alpha C(X, Y) = B\alpha C(X, Y)$. By definition of $wbg\alpha$-closure and $b\alpha$-closure, $wbg\alpha\text{-}cl(E, F) = b\alpha\text{-}cl(E, F)$, for each subset (E, F) of (X, Y, M).

Theorem 2.19. *If* (X, Y, M) *is a* ${}_{wbg\alpha}T_{bc}$*-space, then for each* $(x, y) \in (X, Y)$ *either* $\{(x, y)\}$ *is* $b\alpha$*-closed or open.*

Proof. Let $(x, y) \in (X, Y)$ and suppose $\{(x, y)\}$ is not $b\alpha$-closed in (X, Y, M). Then $(X, Y) - \{(x, y)\}$ is not $b\alpha$-open. Hence (X, Y) is the only $b\alpha$-open set containing $(X, Y) - \{(x, y)\}$ which implies $b\alpha\text{-}cl((X, Y) - \{(x, y)\}) \subseteq (X, Y)$. Hence $(X, Y) \supseteq b\alpha\text{-}cl((X, Y) - \{(x, y)\}) \supseteq b\alpha\text{-}cl(b\text{-}int((X, Y) - \{(x, y)\}))$. Thus $(X, Y) - \{(x, y)\}$ is $wbg\alpha$-closed set of (X, Y, M). Since (X, Y, M) is a ${}_{wbg\alpha}T_{bc}$-space, $(X, Y) - \{(x, y)\}$ is binary closed. Thus $\{(x, y)\}$ is binary open in (X, Y, M).

3. Applications of $bg^{\star}\alpha$-Closed Sets

By applying $bg^{\star}\alpha$-closed sets we will define seven new spaces namely ${}_{b\alpha g}T_{bg^{\star}\alpha}$-space, ${}_{g^{\star}\alpha}T_{bc}$-space, ${}_{bg^{\star}\alpha}T_{b\alpha}$-space, ${}_{bg\alpha}T_{bg^{\star}\alpha}$-space, ${}_{wb\alpha g}T_{bg^{\star}\alpha}$-space, ${}_{wbg\alpha}T_{bg^{\star}\alpha}$-space, ${}_{bg^{\star}\alpha}T_{bg^{\star}}$-space and study some of their properties.

Definition 3.1. *A space* (X, Y, M) *is said to be a*

(1) ${}_{bg^{\star}\alpha}T_{bc}$*-space if every* $bg^{\star}\alpha$*-closed set in it is binary closed.*
(2) ${}_{bg^{\star}\alpha}T_{b\alpha}$*-space if every* $bg^{\star}\alpha$*-closed set in it is* $b\alpha$*-closed.*
(3) ${}_{bg\alpha}T_{bg^{\star}\alpha}$*-space if every* $bg\alpha$*-closed set in it is* $bg^{\star}\alpha$*-closed.*
(4) ${}_{b\alpha g}T_{bg^{\star}\alpha}$*-space if every* $b\alpha g$*-closed set in it is* $bg^{\star}\alpha$*-closed.*

(5) ${}_{\mathrm{wb\alpha g}}T_{\mathrm{bg^{\star}\alpha}}$*-space if every wbαg-closed set in it is bg*α-closed.*

(6) ${}_{\mathrm{wbg\alpha}}T_{\mathrm{bg^{\star}\alpha}}$*-space if every wbgα-closed set in it is bg*α-closed.*

(7) ${}_{\mathrm{bg^{\star}\alpha}}T_{\mathrm{bg^{\star}}}$*-space if every bg*α-closed set in it is bg*-closed.*

Theorem 3.2. *Every* ${}_{\mathrm{wbg\alpha}}T_{\mathrm{bc}}$*-space is also a* ${}_{\mathrm{bg^{\star}\alpha}}T_{\mathrm{b\alpha}}$*-space.*

Proof. Assume that (X, Y, M) is a ${}_{\mathrm{wbg\alpha}}T_{\mathrm{bc}}$-space. Let (E, F) be a $bg^{\star}\alpha$-closed set in (X, Y). Then (E, F) is $wbg\alpha$-closed. By the definition of ${}_{\mathrm{wbg\alpha}}T_{\mathrm{bc}}$-space, (E, F) is binary closed. Since every binary closed set is $b\alpha$-closed, (E, F) is $b\alpha$-closed in (X, Y, M). Thus (X, Y, M) is a ${}_{\mathrm{bg^{\star}\alpha}}T_{\mathrm{b\alpha}}$-space.

The converse of the above Theorem need not be true as seen from the following example.

Example 3.3. *In Example 2.8, then* (X, Y, M) *is a* ${}_{\mathrm{bg^{\star}\alpha}}T_{\mathrm{b\alpha}}$*-space but not a* ${}_{\mathrm{wbg\alpha}}T_{\mathrm{bc}}$*-space, since the subset* $(\{b\}, Y)$ *is wbgα-closed but not binary closed in* (X, Y, M).

Theorem 3.4. *Every binary* ${}_{\mathrm{b\alpha g}}T_{\mathrm{bg^{\star}\alpha}}$*-space is a* ${}_{\mathrm{bg\alpha}}T_{\mathrm{bg^{\star}\alpha}}$*-space.*

Proof. Let (X, Y, M) be a ${}_{\mathrm{b\alpha g}}T_{\mathrm{bg^{\star}\alpha}}$-space and let (E, F) be a $bg\alpha$-closed set in (X, Y). Then (E, F) is $b\alpha g$-closed. Since (X, Y, M) is a ${}_{\mathrm{b\alpha g}}T_{\mathrm{bg^{\star}\alpha}}$-space, (E, F) is $bg^{\star}\alpha$-closed. Hence (X, Y, M) is a ${}_{\mathrm{bg\alpha}}T_{\mathrm{bg^{\star}\alpha}}$-space.

The converse of the above Theorem need not be true as seen from the following example.

Example 3.5. *Let* $X = \{a, b\}$, $Y = \{1, 2\}$ *and* $\mathrm{M} = \{(\phi, \phi), (\phi, \{1\}), (\{a\}, \{1\}), (X, Y)\}$. *Then* (X, Y, M) *is a* ${}_{\mathrm{bg\alpha}}T_{\mathrm{bg^{\star}\alpha}}$*-space but not a* ${}_{\mathrm{b\alpha g}}T_{\mathrm{bg^{\star}\alpha}}$*-space, since the subset* $(\{b\}, \{1\})$ *is bαg-closed but not bg*α-closed in* (X, Y, M).

Theorem 3.6. *If* (X, Y, M) *is a* ${}_{\mathrm{bg^{\star}\alpha}}T_{\mathrm{b\alpha}}$*-space and an binary* T_{α}*-space, then it is* ${}_{\mathrm{bg^{\star}\alpha}}T_{\mathrm{bc}}$*-space.*

Proof. Let (E, F) be a $bg^\star\alpha$-closed set in (X, Y, M). Since (X, Y, M) is a ${}_{bg^\star\alpha}T_{b\alpha}$-space, (E, F) is $b\alpha$-closed in (X, Y, M). Since (X, Y, M) is an binary T_α-space, every $b\alpha$-closed set is binary closed and hence (E, F) is binary closed in (X, Y, M). Hence (X, Y, M) is a ${}_{bg^\star\alpha}T_{bc}$-space.

Theorem 3.7. *If (X, Y, M) is a binary T_α-space and a ${}_{bg^\star\alpha}T_{b\alpha}$-space, then it is a ${}_{bg^\star\alpha}T_{bg^\star}$ -space.*

Proof. Let (E, F) be a $bg^\star\alpha$-closed set in (X, Y, M). Since (X, Y, M) is a ${}_{bg^\star\alpha}T_{b\alpha}$-space, (E, F) is $b\alpha$-closed in (X, Y, M). Since (X, Y, M) is a $b\alpha$-space, every $b\alpha$-closed set is binary closed and hence (E, F) is binary closed in (X, Y, M). Since every binary closed set is $bg^\star$-closed, (E, F) is $bg^\star$-closed in (X, Y, M). Hence (X, Y, M) is a ${}_{bg^\star\alpha}T_{bg^\star}$-space.

Theorem 3.8. *If (X, Y, M) is a binary $T^\star_{1/2}$-space and a ${}_{bg^\star\alpha}T_{bg^\star}$-space, then it is a ${}_{bg^\star\alpha}T_{b\alpha}$-space.*

Proof. Let (E, F) be a $bg^\star\alpha$-closed set in (X, Y, M). Since (X, Y, M) is a ${}_{bg^\star\alpha}T_{bg^\star}$-space, (E, F) is $bg^\star$-closed in (X, Y, M). Since (X, Y, M) is a binary $T^\star_{1/2}$-space, (E, F) is binary closed in (X, Y, M). Since every binary closed set is $b\alpha$-closed, (E, F) is $b\alpha$-closed in (X, Y, M). Hence (X, Y, M) is a ${}_{bg^\star\alpha}T_{b\alpha}$-space.

Theorem 3.9. *If (X, Y, M) is a ${}_{bg^\star\alpha}T_{bc}$-space, then for each $(x, y) \in (X, Y)$ either $\{(x, y)\}$ is $b\alpha$-closed or open.*

Proof. Let $(x, y) \in (X, Y)$ and suppose $\{(x, y)\}$ is not $b\alpha$-closed in (X, Y). Then $(X, Y) - \{(x, y)\}$ is not $b\alpha$-open. Hence (X, Y) is the only $b\alpha$-open set containing $(X, Y) - \{(x, y)\}$ which implies $b\alpha\text{-}cl((X, Y) - \{(x, y)\}) \subseteq (X, Y)$. Hence $(X, Y) - \{(x, y)\}$ is a $bg^\star\alpha$-closed in (X, Y, M). Since (X, Y, M) is a ${}_{g^\star\alpha}T_{bc}$-space, $(X, Y) - \{(x, y)\}$ is binary closed. Thus $\{(x, y)\}$ is binary open in (X, Y, M).

Theorem 3.10. *If* (X, Y, M) *is a* ${}_{\mathrm{bg}^{\star}\alpha}T_{\mathrm{bc}}$*-space, then for every subset* (E, F) *of* (X, Y)*,* $bg^{\star}\alpha$*-*$cl(E, F)$ *is* $bg^{\star}\alpha$*-closed in* (X, Y, M)*.*

Proof. By definition, $bg^{\star}\alpha\text{-}cl(E, F) = \bigcap\{(G, H) \subseteq (X, Y) : (E, F) \subseteq (G, H)$ and (G, H) is $bg^{\star}\alpha$-closed in $(X, Y, \mathrm{M})\}$. Since (X, Y, M) is a ${}_{\mathrm{bg}^{\star}\alpha}T_{\mathrm{bc}}$-space, $bg^{\star}\alpha\text{-}cl(E, F)$ is binary closed in (X, Y, M). Since every binary closed set is $bg^{\star}\alpha$-closed, $bg^{\star}\alpha\text{-}cl(E, F)$ is $bg^{\star}\alpha$-closed in (X, Y, M).

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