Second Order Rotatable Designs of Second Type using Balanced Incomplete Block Designs with Unequal Block Sizes

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Summary

Kim [12]proposed second order rotatable designs (SORD) of second type using central composite designs (CCD),in which the positions of axial points are indicated by two numbers (a_1, a_2). In this paper, a new method of construction on SORD of second type using balanced incomplete block designs (BIBD) with unequal block sizes is suggested. It is shown that in some cases this new method leads to SORD of second type with lesser and equal number of designs than those available in the literature.

Keywords: Second order rotatable designs, Second order rotatable designs of second type, Balanced Incomplete block designs.

1. Introduction

Response surface design is a collection of mathematical and statistical techniques useful for analyzing problems where several independent variables influence a dependent variable.The property of rotatability was proposed by Box and Hunter [1] for response surface designs and constructed second order rotatable central composite designs (CCD). Das and Narasimham[8] constructed second order rotatable designs (SORD) using balanced incomplete block designs (BIBD). Das [6] constructed SORD through BIBD with unequal block sizes.Raghavarao[15, 16] constructed symmetrical unequal block arrangements (SUBA) with two unequal block sizes and constructed SORD through incomplete block designs. Tyagi[22] constructed SORD using pairwise balanced designs (PBD).Khuri [11] studied measure of rotatability for response surface designs.Park et al. [14] suggested measure of rotatability for second order response surface designs. Kim [12] extended rotatable CCD with the axial points indicated by two numbers. Victorbabu[23] studied SORD using pair of partially balanced incomplete block designs. Victorbabuand Vasundharadevi[27] studiedmodified second order response surface designs using BIBD.Victorbabu[24]studied modified second order response surface designs, rotatable designs, rotatable designs with equi-spaced doses. Victorbabu andSurekha [25, 26] developedmeasure of rotatability for second order response surface designs using incomplete block designs and BIBD respectively.Rajyalakshmi andVictorbabu[18]developedSORD under tridiagonal correlated structure of errors using BIBD.Jyostna and Victorbabu[9] studied measure of modified rotatability for second degree polynomial using BIBD.Jyostna et al. [10] suggested measure of modified rotatability for second order response surface designs using CCD.Chiranjeevi et al. [5] developed SORD of second type using CCD. Chiranjeevi and Victorbabu[2, 3, 4] studied SORD of second type using BIBD, PBDand SUBA with two unequal block sizes respectively.

Kim and Ko[13] developed slope rotatability of second type using CCD for $2 \leq v \leq 5$ by taking $n_a = 1$. Ravikumar and Victorbabu[19]extended the results of Kim and Ko[13] and developed SOSRD of second type using CCD for $6 \leq v \leq 17$ by taking $n_a = 1$.Ravikumar and Victorbabu[20, 21]studied SRCCD of second type for$2 \leq v \leq 17$ with $2 \leq n_a \leq 4$ and SOSRD of second type using PBD respectively. Victorbabu and Ravikumar[28] developed SOSRD of second type using BIBD.

In this paper, a new method of construction of SORD of second type using BIBD with unequal block sizes is suggested. It is found that in some cases this new method leads to SORD of second type with less number of design points.

2. Conditions for Second Order Rotatable Designs

A general second order response surface design $D=((X_{iu}))$ for fitting

$$Y_u = \beta_0 + \sum_{i=1}^{v} \beta_i X_{iu} + \sum_{i=1}^{v} \beta_{ii} X_{iu}^2 + \sum_{i=1}^{v} \sum_{j=1}^{v} \beta_{ij} X_{iu} X_{ju} + e_u \qquad (2.1)$$

where X_{iu} denotes the level of the i[th] factor (i=1,2,…,v) in the u[th] run (u=1,2,…,N) of the experiment and e_u 'sare uncorrelated random errors with mean zero and variance σ^2. Then D is said to be SORD if the variance of $Y(X_1, X_2, ..., X_v)$ with respect to each of independent variable X_i is only a function of the distance $\left(d^2 = \sum_{i=1}^{v} X_i^2\right)$ of the point $(X_1, X_2, ..., X_v)$ from the origin (centre) of the design.

The general conditions for SORD are as follows[cf. Box and Hunter [1]].

All odd order moments are zero. In other words when at least one odd power X's equal to zero. i.e;

A. $\sum X_{iu}=0, \sum X_{iu}X_{ju}=0, \sum X_{iu}X_{ju}^2=0, \sum X_{iu}X_{ju}X_{ku}=0,$

$\sum X_{iu}^3=0, \sum X_{iu}X_{ju}^3=0, \sum X_{iu}X_{ju}X_{ku}^2=0, \sum X_{iu}X_{ju}X_{ku}X_{lu}=0,$ etc. for $i \neq j \neq k \neq l$;

B. (i) $\sum X_{iu}^2 = \text{constant} = N\lambda_2$

(ii) $\sum X_{iu}^4 = \text{constant} = cN\lambda_4$, for all i

C. $\sum X_{iu}^2 X_{ju}^2 = \text{constant} = N\lambda_4$, for all $i \neq j$ (2.2)

From B (ii) and C of (2.2), we have $\sum X_{iu}^4 = c\sum X_{iu}^2 X_{ju}^2$

where c, λ_2 and λ_4 are constants.

The variances and covariances of the estimated parameters are [cf. Das and Giri[7]]

$$V(\hat{\beta}_0) = \frac{\lambda_4(c+v-1)\sigma^2}{N[\lambda_4(c+v-1)-v\lambda_2^2]}$$

$$V(\hat{\beta}_i) = \frac{\sigma^2}{N\lambda_2}$$

$$V(\hat{\beta}_{ij}) = \frac{\sigma^2}{N\lambda_4}$$

$$V(\hat{\beta}_{ii}) = \frac{\sigma^2}{(c-1)N\lambda_4}\left[\frac{\lambda_4(c+v-2)-(v-1)\lambda_2^2}{\lambda_4(c+v-1)-v\lambda_2^2}\right]$$

$$Cov(\hat{\beta}_0, \hat{\beta}_{ii}) = \frac{-\lambda_2\sigma^2}{N[\lambda_4(c+v-1)-v\lambda_2^2]}$$

$$Cov(\hat{\beta}_{ii}, \hat{\beta}_{jj}) = \frac{(\lambda_2^2-\lambda_4)\sigma^2}{(c-1)N\lambda_4[\lambda_4(c+v-1)-v\lambda_2^2]}$$ and other covariances vanish. (2.3)

An inspection of the $V(\hat{\beta}_0)$ shows that a necessary condition for the existence of a non singular second order design is

D. $\frac{\lambda_4}{\lambda_2^2} > \frac{v}{c+v-1}$ (Non-singularity condition) (2.4)

The variance of the estimated response at the point is $\hat{Y}$

$$V(\hat{Y}) = V(\hat{\beta}_0) + \left[V(\hat{\beta}_i) + 2Cov(\hat{\beta}_0, \hat{\beta}_{ii})\right]d^2 + V(\hat{\beta}_{ii})d^4 + \sum X_{iu}^2 X_{ju}^2 \left[V(\hat{\beta}_{ij}) + 2Cov(\hat{\beta}_{ii}, \hat{\beta}_{jj}) - 2V(\hat{\beta}_{ii})\right] \quad (2.5)$$

The coefficient of $\sum X_{iu}^2 X_{ju}^2$ in the above equation (2.5) is simplified to $(c-3)\sigma^2/(c-1)N\lambda_4$.

A second order response surface design D is said to be rotatable, if in this design c=3 and all other conditions (2.2) to (2.4) are satisfied.

3. New Method of Construction on Second Order Rotatable Designs of Second Type usingBalanced Incomplete Block Designs with Unequal Block Sizes

Das [6] constructed SORD through BIBD with unequal block sizes.Kim [12] developed second type of rotatable CCD, in which the positions of axial points are indicated by two numbers a_1 and a_2 ($a_2 \geq a_1 > 0$) for $2 \leq v \leq 8$. Chiranjeevi et al. [5]extended the results of Kim [12] and developedSORD of second type using CCD for $9 \leq v \leq 17$. Chiranjeevi and Victorbabu[2, 3, 4]developedSORD of second type using BIBD, PBD and SUBAwith two unequal block sizes respectively.

Here, another new method of constructing SORD of second type using BIBD with unequal block sizes is suggested.A BIBD design with unequal block sizes $(v, b, r, k_1, k_2, ..., k_m, \lambda)$ is defined as an arrangement of v-treatments in b blocks of sizes $k_1, k_2, ..., k_m$ such that each treatment occurs exactly in r blocks and any pair of treatments occurs in exactly λ blocks. If b_i is the number of blocks of size k_i, then we have in these designs,

$$vr = \sum_{i=1}^{m} b_i k_i \quad ; \qquad \lambda v(v-1) = \sum_{i=1}^{m} b_i k_i (k_i - 1)$$

For the purpose of constructing SORD of second type we take a particular class of these designs in which the replication of every treatment is a constant r_i in the set of b_i blocks each of size k_i for all i. For this class of designs we must further have

$vr_i = b_i k_i$ and $\sum r_i = r$

By taking a BIBD (v, b, r, k, λ), a particular treatment is omitted, we shall get another BIBD with unequal block sizes $(v\text{-}1, b, r, k, k\text{-}1, \lambda)$ such that in (b-r) blocks each of size k each treatment will be replicated (r-λ) times, while in the remaining r blocks each of size (k-1) each treatment will be replicated λ times.

Through BIBD with unequal block sizes, SORD of second type can be obtained as follows. Let us write the BIBD with unequal block sizes $(v\text{-}1, b, r, k_1, k_2, ..., k_m, \lambda)$ as a bxv matrix with elements zero (when a treatment does not appear in a block) and unity (when the treatment appears) in the set of b_1 blocks of size k_1, zero and α_1 (when the treatment appears) in the set of b_2 blocks of size k_2, zero and α_2 (when the treatment appears) in the set of b_3 blocks of size k_3 and so on.

The design plan of SORD of second type using BIBD with unequal block sizes in which the position of the axial points are indicated by two numbers a_1 and a_2 $(a_2 \geq a_1 > 0)$. Let $(v, b_1, b_2, r_1, r_2, k_1, k_2, \lambda)$ denote the parameters of BIBD with two unequal block sizes, $2^{t(k_i)}$ denote a fractional replicate of 2^{k_i} in ±1 levels, in which no interaction with less than five factors is confounded and n_0 be the number of central points in the design.Let $\lceil\lfloor 1\text{-}(v,b_1,r_1,k_1,\lambda)\rceil\rfloor$ and $\lceil\lfloor \alpha\text{-}(v,b_2,r_2,k_2,\lambda)\rceil\rfloor$ are denotes the design points generated from the transpose of incidence matrix of BIBD with two unequal block sizes. Let $\left[1-(v, b_1, r_1, k_1, \lambda)\right]2^{t(k_1)}$ are the $b_1 2^{t(k_1)}$ and $\left[\alpha\text{-}(v, b_2, r_2, k_2, \lambda)\right]2^{t(k_2)}$ are the $b_2 2^{t(k_1)}$ design points generated from BIBD with two unequal block sizes by multiplication (cf.Raghavarao[17]). We use the additional set of points like $(\pm a_1,0,...,0),(0,\pm a_1,0,...,0),...,(0,0,...,\pm a_1)$; $(\pm a_2,0,...,0),(0,\pm a_2,0,...,0),...,(0,0,...,\pm a_2)$ are two sets of axial points. Here $(a_1,0,...,0)2^1 U (a_2,0,...,0)2^1$ denote the 4v axial points generated from $(a_1,0,...,0)$ and $(a_2,0,...,0)$ point sets. Let U denote the union of the design points generated from different sets of points and n_0 denote the number of central points. The new method of construction of SORD of second type using BIBD with two unequal block sizes is given in the following theorem (cf. Kim [12], Chiranjeevi and Victorbabu [2, 3, 4]).

Theorem (3.1):

If $(v, b_1, b_2, r_1, r_2, k_1, k_2, \lambda)$ is a BIBD with two unequal block sizes and n_0 is the number of pre-fixed central points, the design points,

$[1\text{-}(v, b_1, r_1, k_1, \lambda)]^* \times 2^{t(k_1)} U [\alpha\text{-}(v, b_2, r_2, k_2, \lambda)]^* \times 2^{t(k_2)} U (a_1, 0, ..., 0) 2^1 U (a_2, 0, ..., 0) 2^1 U (n_0)$

willgive a v-dimensional SORD of second type using BIBD with two unequal block sizes in

$N = b_1 2^{t(k_1)} + b_2 2^{t(k_2)} + 4v + n_0$ design points, with $a_1^4 + a_2^4 = (3\lambda - r_1 - r_2) 2^{t(k_1)-1}$.

Proof: For the design points generated from SORD of second type using BIBD with two unequal block sizes, simple symmetry conditions A, B and C of equation (2.2) are true. Condition (A) of equation (2.2) is true obviously, condition (B) and (C) of (2.2) are true as follows.

B. (i) $\sum X_{iu}^2 = r_1 2^{t(k_1)} + r_2 2^{t(k_2)} \alpha^2 + 2a_1^2 + 2a_2^2 = N\lambda_2$ (3.1)

(ii) $\sum X_{iu}^4 = r_1 2^{t(k_1)} + r_2 2^{t(k_2)} \alpha^4 + 2a_1^4 + 2a_2^4 = 3N\lambda_4$ (3.2)

C. $\sum X_{iu}^2 X_{ju}^2 = \lambda 2^{t(k_1)} = \lambda 2^{t(k_2)} \alpha^4 = N\lambda_4$ (3.3)

From (3.3) we have $\alpha^4 = 2^{t(k_1)-t(k_2)}$

Solving the equations (3.2) and (3.3),

$\Rightarrow r_1 2^{t(k_1)} + r_2 2^{t(k_1)} + 2\left(a_1^4 + a_2^4\right) = 3\lambda 2^{t(k_1)}$ $\left[\alpha^4 = 2^{t(k_1)-t(k_2)}\right]$

$\Rightarrow a_1^4 + a_2^4 = (3\lambda - r_1 - r_2) 2^{t(k_1)-1}$

Example (i):

We illustrate the use of theorem (3.1) by constructing a SORD of second type for 5-factors using BIBD with two unequal block sizes.

Here we take the help of BIBD with two unequal block sizes for v=5 obtained by deleting a treatment from the BIBD (v=6, b=10, r=5, k=3, λ=2). The design points,

$[1\text{-}(v=5, b_1=5, r_1=3, k_1=3, \lambda=2)] 2^{t(3)} U [\alpha\text{-}(v=5, b_2=5, r_2=2, k_2=2, \lambda=2)] 2^{t(2)} U (a_1, 0, ..., 0) 2^1 U (a_2, 0, ..., 0) 2^1 U (n_0)$

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* $[1\text{-}(v, b_1, r_1, k_1, \lambda)]^*$ and $[\alpha\text{-}(v, b_2, r_2, k_2, \lambda)]^*$ are the design points generated from the blocks of sizes k_1 and k_2 respectively from the given BIBD with unequal block sizes.

will give a SORD of second type using BIBD with two unequal block sizes in N=81 design points with $n_0=1$ and $a_1=1$.

For the design points generated from SORD of second type using BIBD with two unequal block sizes, simple symmetry conditions A of equation (2.2) are true.

Here B and C of equations(3.1), (3.2) and (3.3) are

B. (i) $\sum X_{iu}^2 = 24+8\alpha^2+2a_1^2+2a_2^2 = N\lambda_2$

(ii) $\sum X_{iu}^4 = 24+8\alpha^4+2a_1^4+2a_2^4 = 3N\lambda_4$

C. $\sum X_{iu}^2 X_{ju}^2 = 16 = 8\alpha^4 = N\lambda_4$ (3.4)

From B (ii) and C of equation (3.4), we have $a_1^4+a_2^4=4$

We assume an arbitrary value $a_1=1$, then we get $a_2=1.3161$. It can be verified that non singularity condition D of (2.4) is satisfied.

It may be point out here that this SORD of second type using BIBD with two unequal block sizes has only 81 design points for 5 factors, whereas the corresponding SORD of second type obtained usingBIBD of Chiranjeevi and Victorbabu[2]for 5factors need101 design points.

Example (ii):

Construction of SORD of second type for 10-factors using BIBD with two unequal block sizes.

Here we take the help of BIBD with two unequal block sizes for v=10 obtained by deleting a treatment from the BIBD (v=11, b=11, r=5, k=5, λ=2). The design points,

$[1-(v=10,b_1=6,r_1=3,k_1=5,\lambda=2)]2^{t(4)}U[\alpha-(v=10,b_2=5,r_2=2,k_2=4,\lambda=2)]2^{t(4)}U(a_1,0,...,0)2^1U(a_2,0,...,0)2^1U(n_0)$

will give a SORD of second type using BIBD with two unequal block sizes in N=217 design points with $n_0=1$ and $a_1=1$.

For the design points generated from SORD of second type using BIBD with two unequal block sizes, simple symmetry conditions A of equation (2.2) are true.

Here B and C of equations(3.1), (3.2) and (3.3) are

B. (i) $\sum X_{iu}^2 = 48+32\alpha^2+2a_1^2+2a_2^2 = N\lambda_2$

(ii) $\sum X_{iu}^4 = 48+32\alpha^4+2a_1^4+2a_2^4 = 3N\lambda_4$

C. $\sum X_{iu}^{2} X_{ju}^{2} = 32 = 32\alpha^{4} = N\lambda_{4}$ (3.5)

From B (ii) and C of equation (3.5), we have $a_1^4 + a_2^4 = 8$

We assume an arbitrary value $a_1 = 1$, then we get $a_2 = 1.6266$. It can be verified that non singularity condition D of (2.4) is satisfied.

In the case of 10-factors, this new method needs 217 design points,whereas the corresponding SORD of second type obtained using BIBD of Chiranjeevi and Victorbabu[2], PBD of Chiranjeevi and Victorbabu[3] and SUBA with two unequal block sizes of Chiranjeevi and Victorbabu[4] for 10 factors need 329, 217 and 217 design points, .

Thus the new method SORD of second type using BIBD with two unequal block sizessometimes leads to lesser or equal number of design points than the SORD of second type obtained through BIBD, PBD and SUBA with two unequal block sizes.

The Appendix table gives the appropriate rotatability values of the parameter a_2 with $a_1 = 1$ for designs using a BIBD with unequal block sizes for $3 \le v \le 14$.

Appendix

Table: Values of a_2 taking a_1=1for SORD of second type using BIBD with unequal block sizes for $3 \le v \le 14$

[These are SORDs of second type with design points
$[1\text{-}(v, b_1, r_1, k_1, \lambda)]^* \times 2^{t(k_1)} U [\alpha\text{-}(v, b_2, r_2, k_2, \lambda)]^* \times 2^{t(k_2)} U (a_1, 0, ..., 0) 2^1 U (a_2, 0, ..., 0) 2^1 U (n_0)$]

BIBD design (v,b,r,k,λ)	BIBD with unequal block sizes $(v,b_1,r_1,k_1,\lambda),(v,b_2,r_2,k_2,\lambda)$	n_0	N	$a_1^4+a_2^4$	a_2
(4,4,3,3,2)	(3,1,1,3,2),(3,3,2,2,2)	1	33	12	1.8212
(5,5,4,4,3)	(4,1,1,4,3),(4,4,3,3,3)	1	65	40	2.4990
(5,10,6,3,3)	(4,4,3,3,3),(4,6,3,2,3)	1	73	12	1.8212
(6,10,5,3,2)	(5,5,3,3,2),(5,5,2,2,2)	1	81	4	1.3161
(6,6,5,5,4)	(5,1,1,5,4),(5,5,4,4,4)	1	117	56	2.7233
(6,15,10,4,6)	(5,5,4,4,6),(5,10,6,3,6)	1	181	64	2.8173
(7,7,4,4,2)	(6,3,2,4,2),(6,4,2,3,2)	1	105	16	1.968
(8,14,7,4,3)	(7,7,4,4,3),(6,4,2,3,2)	1	197	16	1.968
(9,18,8,4,3)	(8,10,5,4,3),(8,8,3,3,3)	1	257	8	1.6266
(9,12,8,6,5)	(8,4,3,6,5),(8,8,5,5,5)	1	289	112	3.2459
(9,18,10,5,5)	(8,8,5,5,5),(8,10,5,4,5)	1	321	40	2.499
(10,18,9,5,4)	(9,9,5,5,4),(9,9,4,4,4)	1	325	24	2.1899
(11,11,5,5,2)	(10,6,3,5,2),(10,5,2,4,2)	1	217	8	1.6266
(11,11,6,6,3)	(10,5,3,6,3),(10,6,3,5,3)	1	297	48	2.6183
(12,22,11,6,5)	(11,11,6,6,5),(11,11,5,5,5)	1	573	64	2.8173
(13,26,12,6,5)	(12,14,7,6,5),(12,12,5,5,5)	1	689	48	2.6183
(15,15,7,7,3)	(14,8,4,7,3),(14,7,3,6,3)	1	793	64	2.8173

* $[1\text{-}(v, b_1, r_1, k_1, \lambda)]^*$ and $[\alpha\text{-}(v, b_2, r_2, k_2, \lambda)]^*$ are the design points generated from the blocks of sizes k_1 and k_2 respectively from the given BIBD with unequal block sizes.

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